

# How to Measure Beam Neutralization in the Accumulator

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## 1 Introduction

This report addresses the question of how to measure the beam neutralization in the Accumulator. Positive ions created from collisions between the residual gas and the antiproton beam can be trapped in the potential of the beam thereby neutralizing the beam. These ions then interact electromagnetically with the beam producing a tune shift. The quantity of interest is the neutralization factor  $N$  defined by

$$N = \frac{N_{ion}}{N_{\bar{p}}} \quad (1)$$

where  $N_{ion}$  is the number of trapped ions and  $N_{\bar{p}}$  is the number of antiprotons.  $N$  will vary between  $N = 0.0$  representing no trapped ions and thereby no neutralization and  $N = 1.0$  representing full neutralization of the beam.

As described by Y. Baconnier in The Proceedings of the 1984 CERN Accelerator School (CERN 85-19), the force on an antiproton is given by

$$F = eE(1/\gamma^2 - N) \quad (2)$$

where  $E$  is the electric field of the antiprotons. The  $1/\gamma^2$  term is the space charge term of the antiprotons. The magnetic field contribution due to the circulating antiprotons is of opposite sign to the static electric field thereby giving the  $1/\gamma^2$  factor. Also note that we are assuming the ions will have the same transverse distribution as that of the antiprotons.

In particular for a round Gaussian beam, the force is given by

$$F = \frac{e^2 N_{\bar{p}l}}{2\pi\epsilon_0 r} [1 - e^{r^2/2\sigma^2}] [1/\gamma^2 - N] \quad (3)$$

where  $N_{\bar{p}l}$  is the number of antiprotons per unit length,  $r$  is the transverse radius of the particle, and  $\sigma$  is the variance of the Gaussian. The resulting tune shift for small amplitude particles is given by

$$\Delta Q = \frac{N_{\bar{p}r_o}}{4\pi\sigma^2\gamma\beta} [N - 1/\gamma^2] \quad (4)$$

where  $N_{\bar{p}}$  is the number of antiprotons in the ring and  $r_o$  is the classical radius of the proton. This represents the maximum tune shift because particles of larger amplitude will experience less force and therefore have smaller tune shifts.

One possible way of measuring this tune shift is by comparing the Betatron side bands of a Transverse Schottky signal for a beam where all the ion clearing electrodes are turned off with the Betatron side bands of a Transverse Schottky signal for a beam where the electrodes are all on. The assumption here, of course, is that one assumes no neutralization for the case where the electrodes are on. The rest of this report will focus on whether one can determine the Neutralization factor  $N$  by comparing the two resulting spectra.

## 2 Calculation

The program TPOT was used to track particles through the Accumulator lattice. Steve Stahl's version of TPOT was modified to include the neutralization effect. The program creates a two dimensional Gaussian distribution of particles in x and y and then tracks them through the lattice. The neutralization plus space charge effect is simulated by giving a kick to the particles at the three low and the three high dispersion points of the lattice. Each kick represents the sum of the effect for the previous sixth of the lattice. The magnitude of the kick for a given particle is determined from the particle's position at the kick point and the E field due to the particle distribution at the kick point.

At one of the zero dispersion points the position of each particle is written to disk. After TPOT has finished the Floquet coordinates of each particle is

then computed from the TPOT generated Twiss parameters of the lattice. The fractional part of the tune of a given particle for a given turn is then calculated from the phase advance from the previous turn.

Figures 1-15 are y tune distributions for runs with different values of the Neutralization factor N. Each particle has been weighted by its "emittance" with the whole distribution then renormalized to the total number of particles. This distribution should then be like the power distribution that one gets from a spectrum analyzer of a transverse Schottky signal. The beam current is 100 mAmp and the emittance is  $2\pi$  mm-mrad. Figures 16-30 are semilog plots of the same distributions. All figures have a bin size in tune of 0.0002 unless noted on the plot. Table 1 summarizes the mean and variance of these distributions.

Table 2 summarizes the means, peaks, and maximum tunes of these distributions. The peaks are my estimation of what they are. The maximums are determined by integrating the histograms until the given percentage (either 99% or 99.9%) of total is reached. Figures 31-34 are plots of these tunes versus the Neutralization factor N.

The averages appear to be linear in N. The peaks are not as nice. The distributions are non-Gaussian and asymmetric with an increasing tail at higher tune and an increasing width as N increases. Estimating the peaks without fitting to a known distribution is therefore, given the statistics, somewhat subjective. Also given the distributions the peaks need not be linear in N.

The maximums appear to be linear above  $N = 0.20$ . Below that one gets into the smear caused by the momentum distribution and the discreetness of the calculation. The two maximums also have different slopes. The question therefore arises what is the true "maximum". Equation 4 of the introduction gives a value of 0.0399.

But how do these slopes vary with the Current I and the Emittance E? Table 3 and Figure 35 show how they vary with the Current I. Table 4 and Figure 36 show how they vary with the Emittance E. As can be seen, the slopes are linear with the current I and with  $1/E$  as expected from Equation 4.

### **3 Conclusions**

It appears like the best way to measure the Neutralization factor N within the context of this model is to read out a betatron sideband distribution of a Schottky spectrum and do a power average of the distribution. One would first subtract off the noise floor, then obtain the average frequency by weighting each frequency point by its power (linear scale), and then convert this frequency to its corresponding tune.

### **4 Tables and Figures**

Neutralization N	y		x	
	Average <sub>y</sub>	Variance $\sigma_y$	Average <sub>x</sub>	Variance $\sigma_x$
0.00	0.619427	0.001012	0.618247	0.001589
0.01	0.619592	0.001009	0.618387	0.001585
0.03	0.619923	0.001015	0.618666	0.001583
0.05	0.620253	0.001037	0.618946	0.001588
0.08	0.620749	0.001097	0.619364	0.001611
0.10	0.621079	0.001153	0.619644	0.001635
0.20	0.622732	0.001565	0.621039	0.001853
0.30	0.624384	0.002098	0.622431	0.002191
0.40	0.626037	0.002684	0.623821	0.002602
0.50	0.627688	0.003297	0.625209	0.003056
0.60	0.629340	0.003926	0.626595	0.003535
0.70	0.630991	0.004566	0.627978	0.004029
0.80	0.632640	0.005214	0.629382	0.004581
0.90	0.634288	0.005866	0.630738	0.005043
1.00	0.635933	0.006523	0.632115	0.005556

Table 1: Average Tunes for 100 mAmp

Neutralization N	Average <sub>y</sub>	Peak <sub>y</sub>	Maximum <sub>y</sub> (99%)	Maximum <sub>y</sub> (99.9%)
0.00	0.619427	0.61940	0.62175	0.62235
0.01	0.619592	0.61960	0.62185	0.62245
0.03	0.619923	0.62000	0.62215	0.62275
0.05	0.620253	0.62020	0.62245	0.62305
0.08	0.620749	0.62080	0.62295	0.62365
0.10	0.621079	0.62105	0.62345	0.62405
0.20	0.622732	0.62280	0.62635	0.62745
0.30	0.624384	0.62430	0.62955	0.63125
0.40	0.626037	0.62560	0.63290	0.63550
0.50	0.627688	0.62660	0.63650	0.63950
0.60	0.629340	0.62800	0.63970	0.64370
0.70	0.630991	0.62930	0.64330	0.64770
0.80	0.632640	0.63070	0.64690	0.65170
0.90	0.634288	0.63200	0.65050	0.65610
1.00	0.635933	0.63360	0.65410	0.66010

Table 2: Y Tunes for 100 mAmp

Current I (mamp)	Slope <sub>avg</sub>	Intercept <sub>avg</sub>	Slope <sub>max</sub>	Intercept <sub>max</sub>
080.	0.01321370	0.61946434	0.03139341	0.62029016
100.	0.01651082	0.61942947	0.04048785	0.61943668
120.	0.01979193	0.61939430	0.04870491	0.61945730
140.	0.02306022	0.61936134	0.05714755	0.61922133

Table 3: Intensity Study for  $2\pi$  Emittance. The fits to get the slopes include all the N values in Table 1 for 100 mAmp but only N = 0.00, 0.05, 0.10, 0.50, and 1.00 for the rest.

Emittance E (mm-mrad)	Slope <sub>avg</sub>	Intercept <sub>avg</sub>	Slope <sub>max</sub>	Intercept <sub>max</sub>
$\pi$	0.03067503	0.61930412	0.07755740	0.61853606
$1.5\pi$	0.02141308	0.61938125	0.05267212	0.61920816
$2\pi$	0.01651082	0.61942947	0.04095831	0.61909175
$2.5\pi$	0.01345735	0.61945790	0.03183604	0.62045407
$3\pi$	0.01136998	0.61947680	0.02586882	0.62110329

Table 4: Emittance Study for 100 mAmp Current. The fits to get the slopes include all the N values in Table 1 for 100 mAmp but only N = 0.00, 0.05, 0.10, 0.50, and 1.00 for the rest.

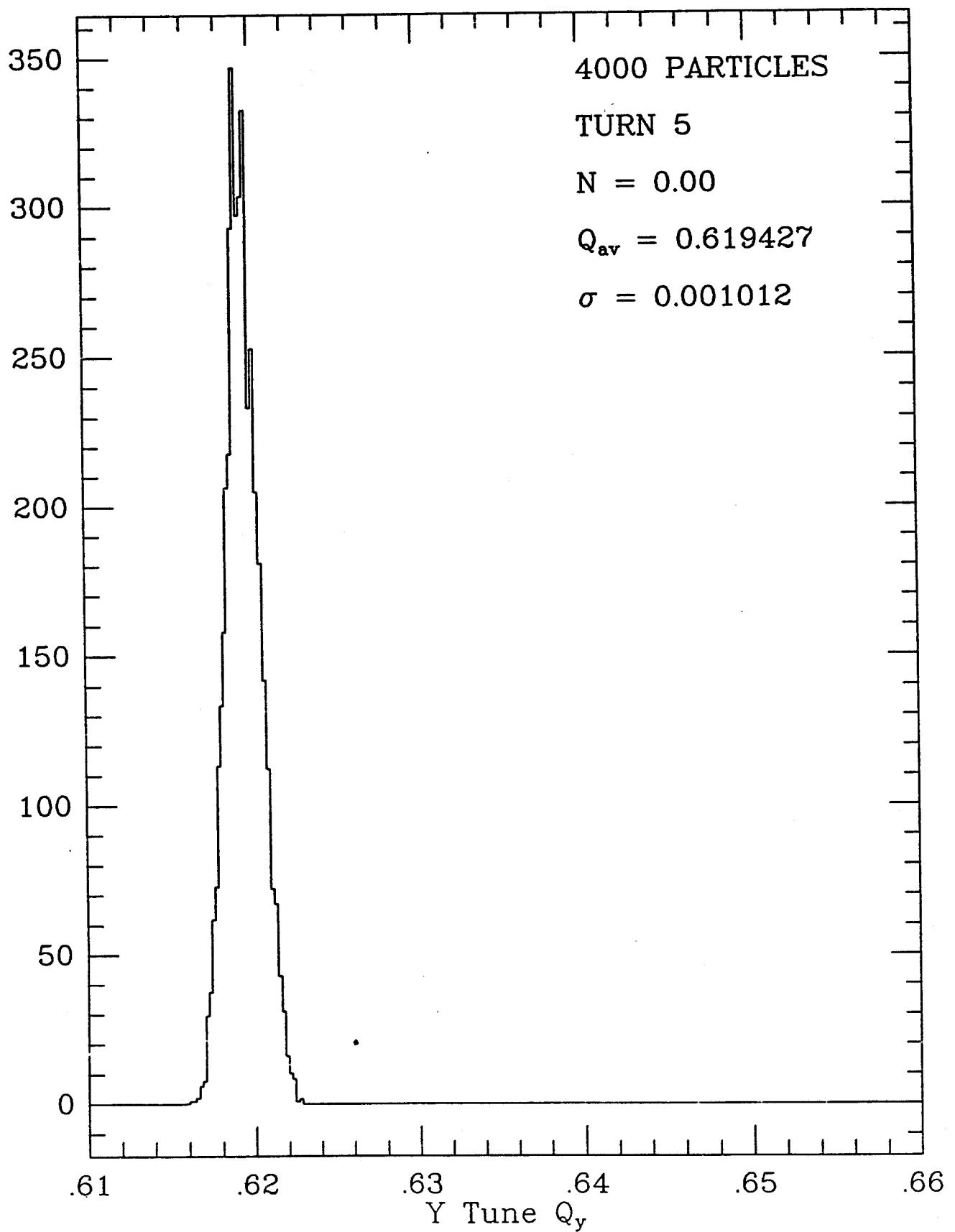


Figure 1

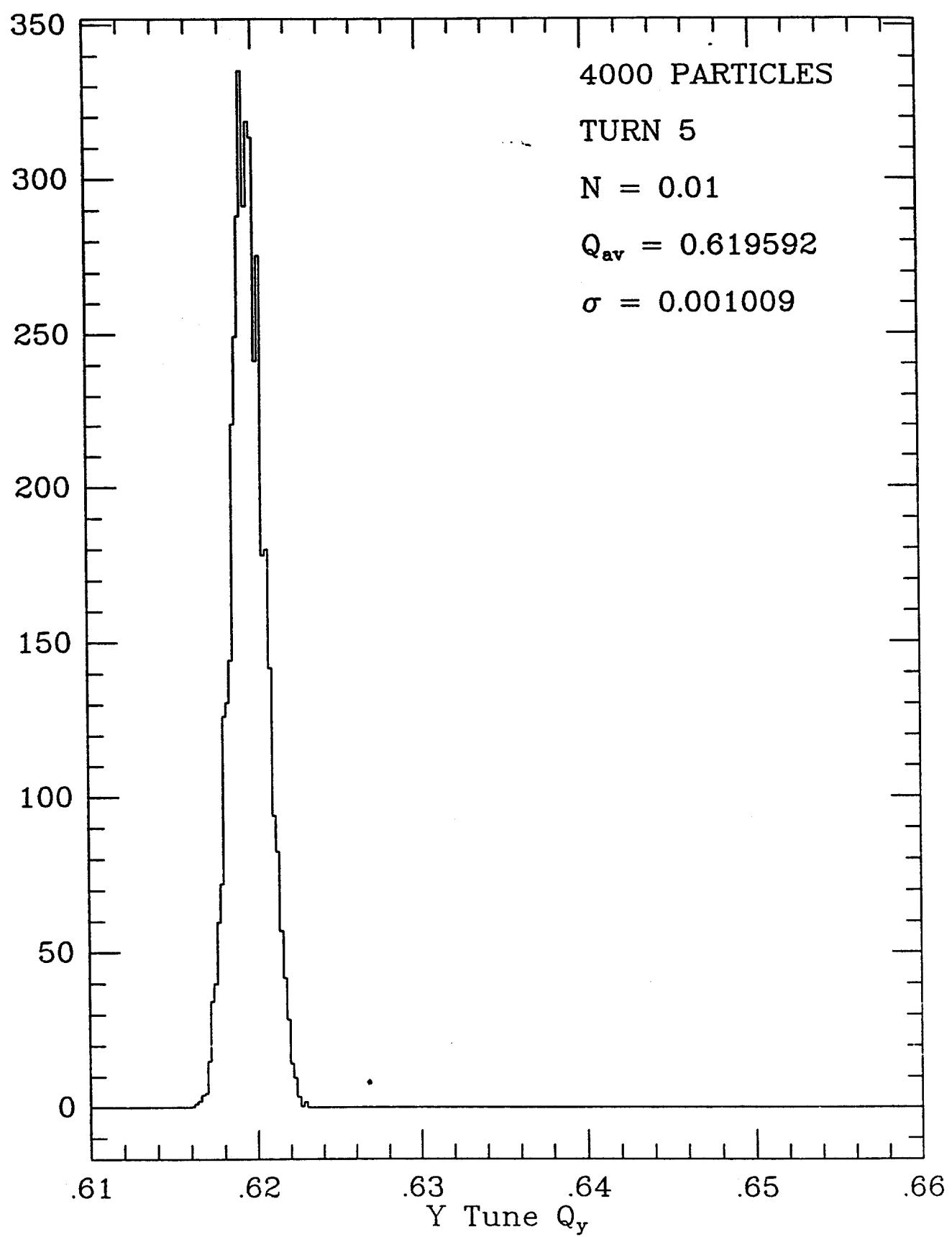


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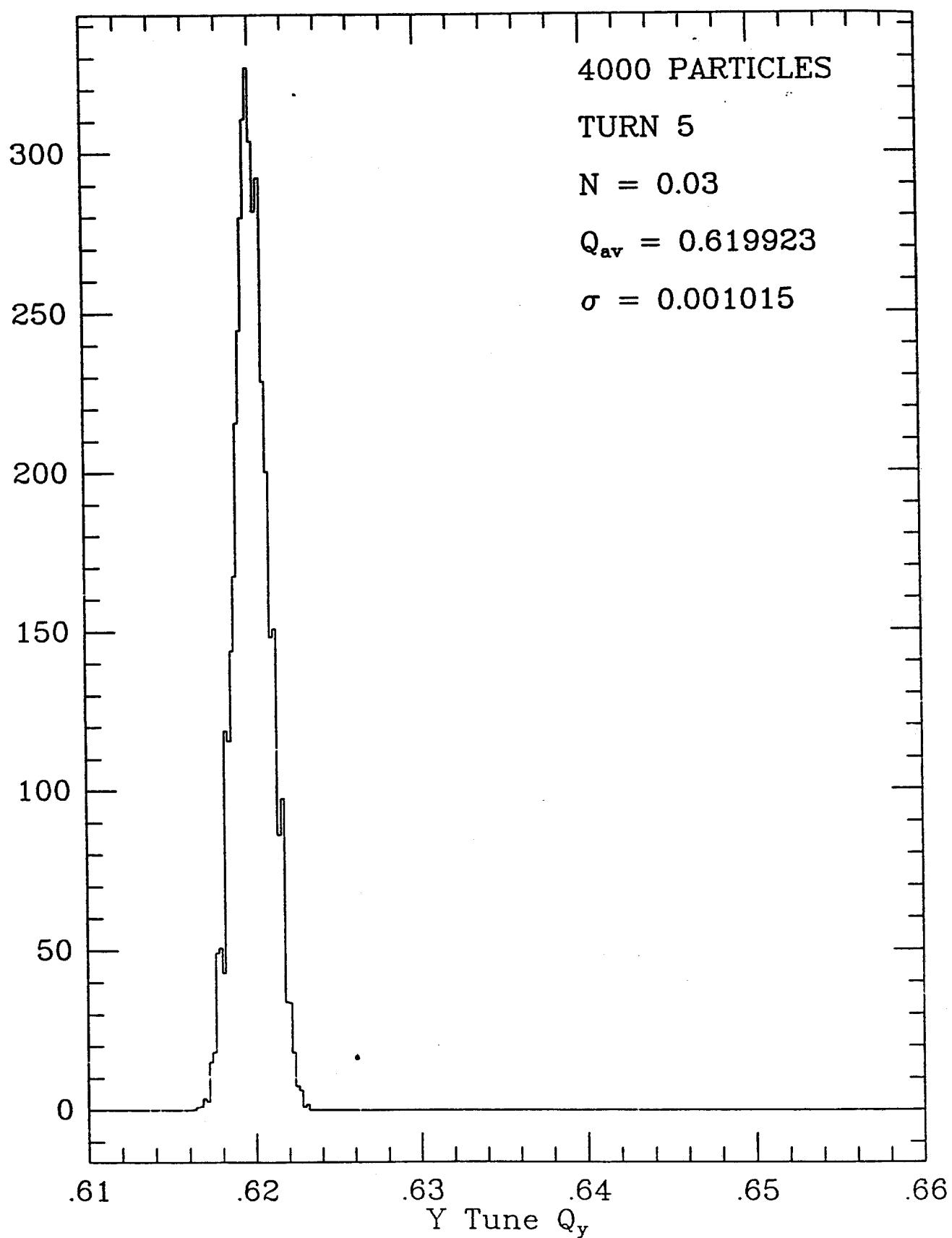


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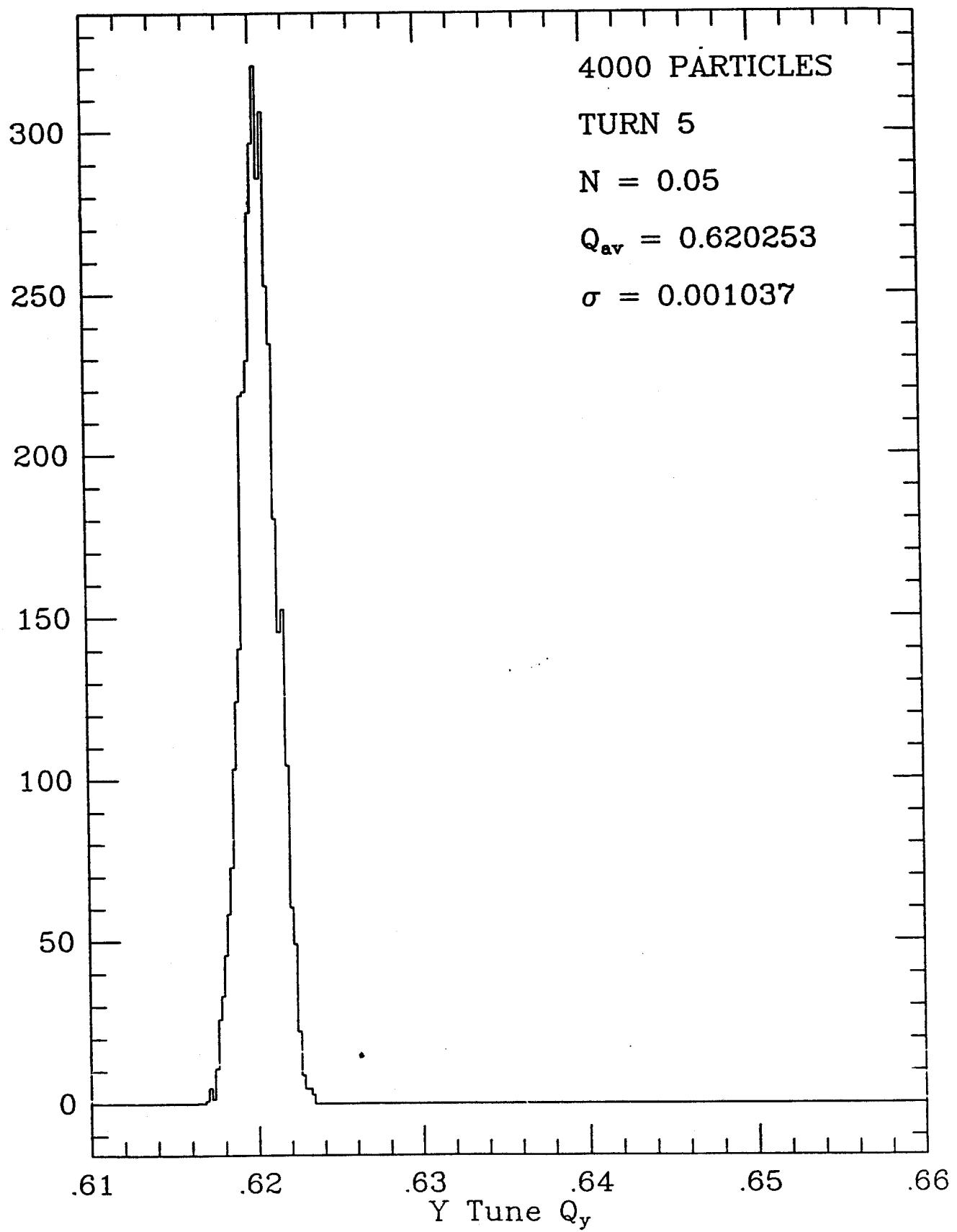


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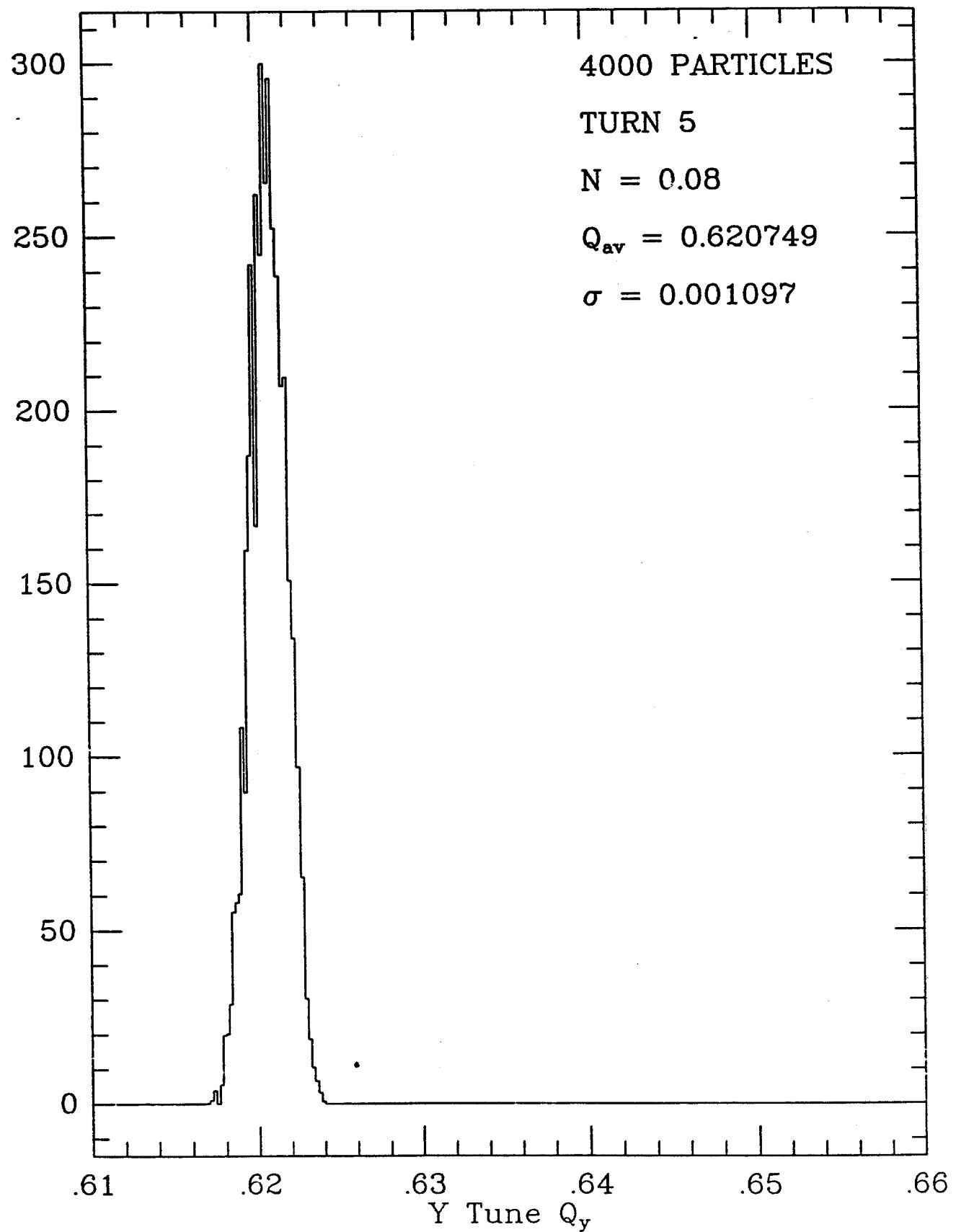


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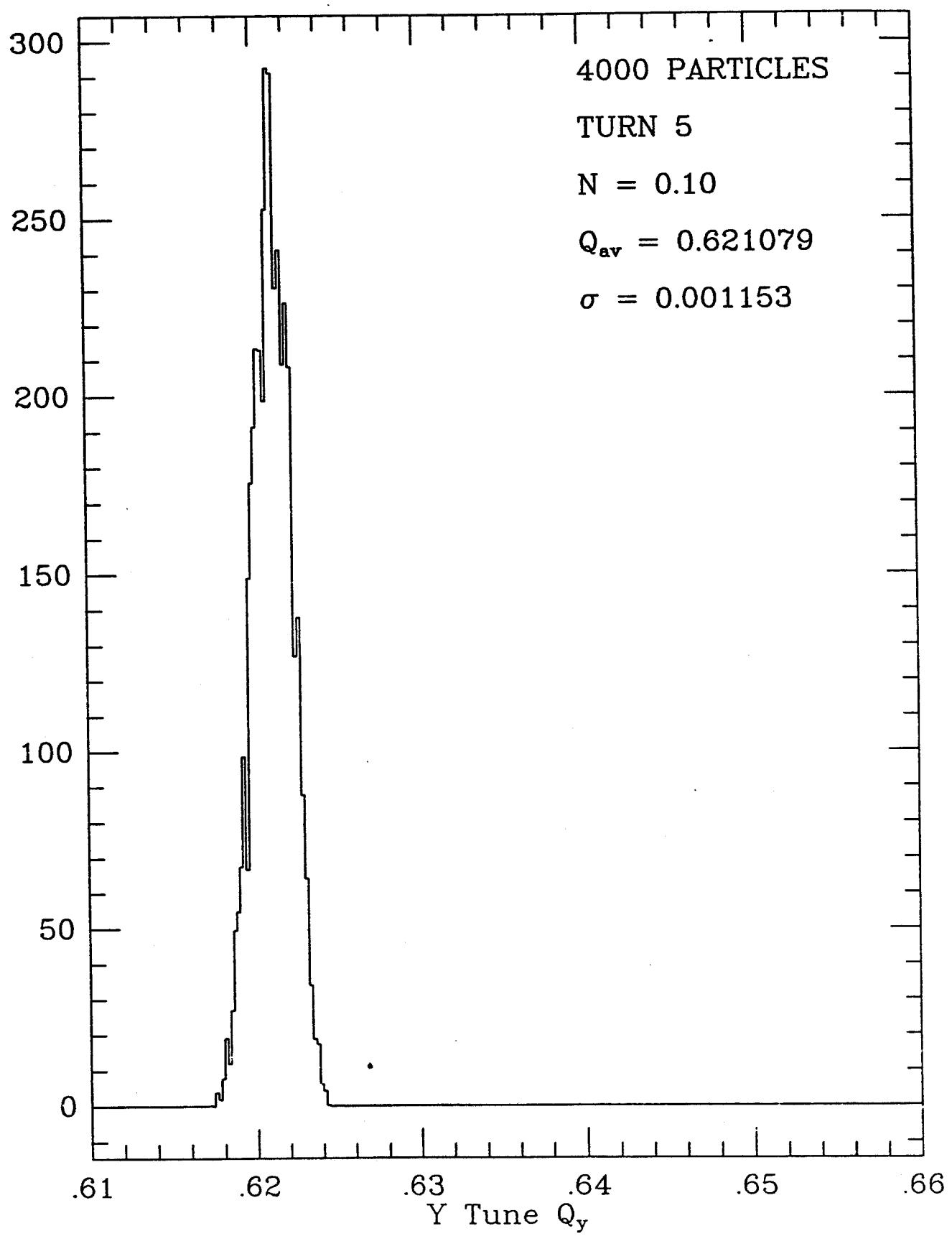


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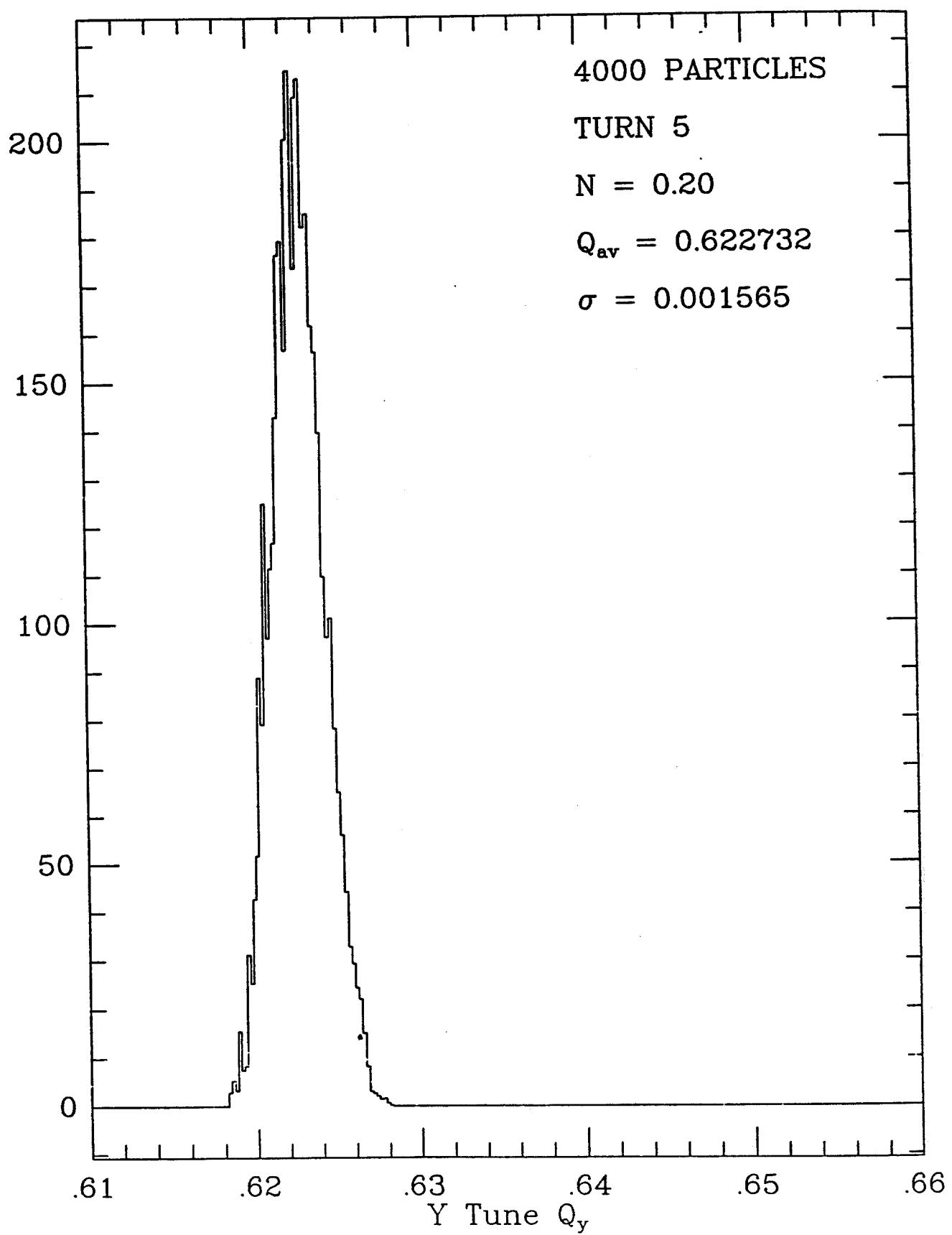


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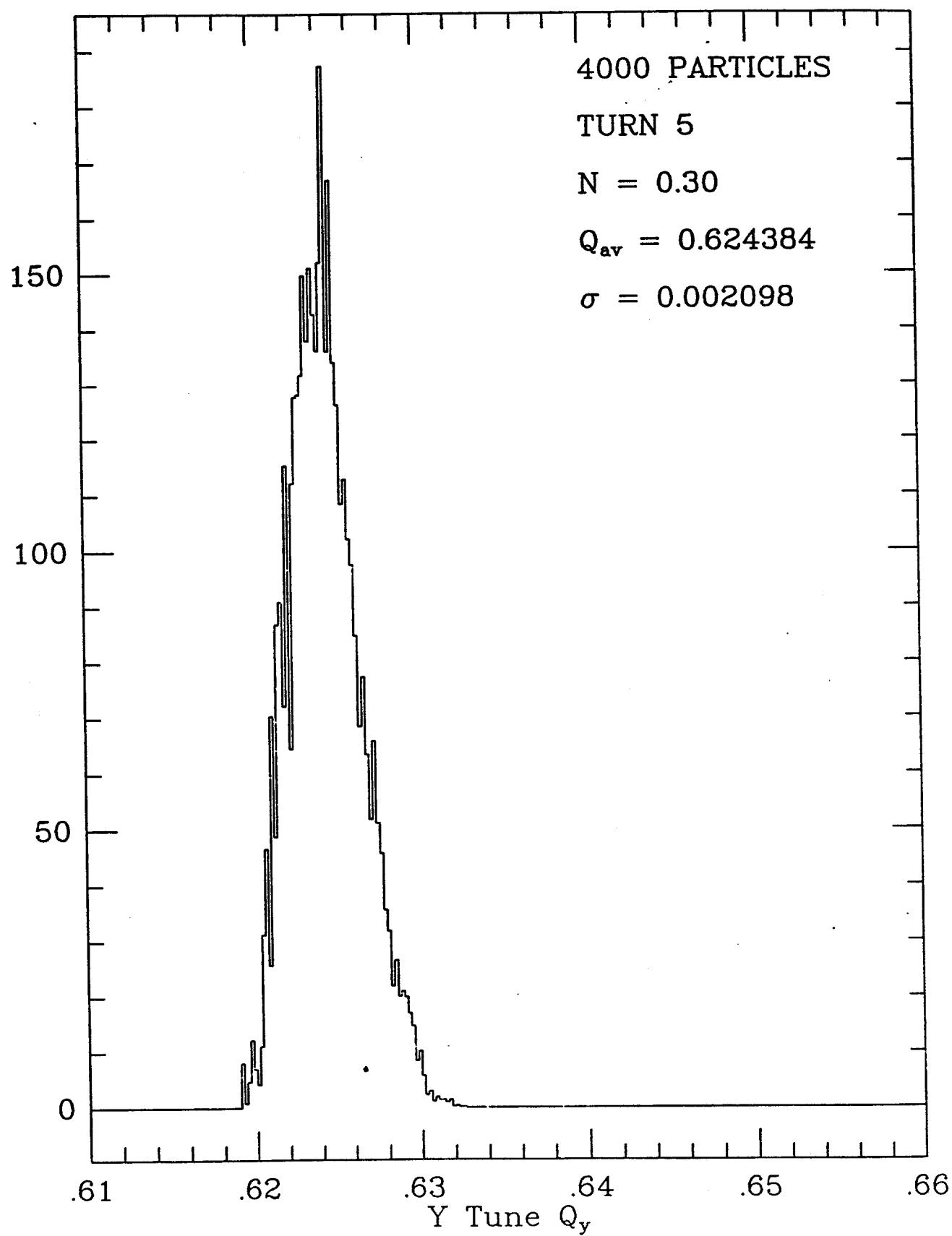


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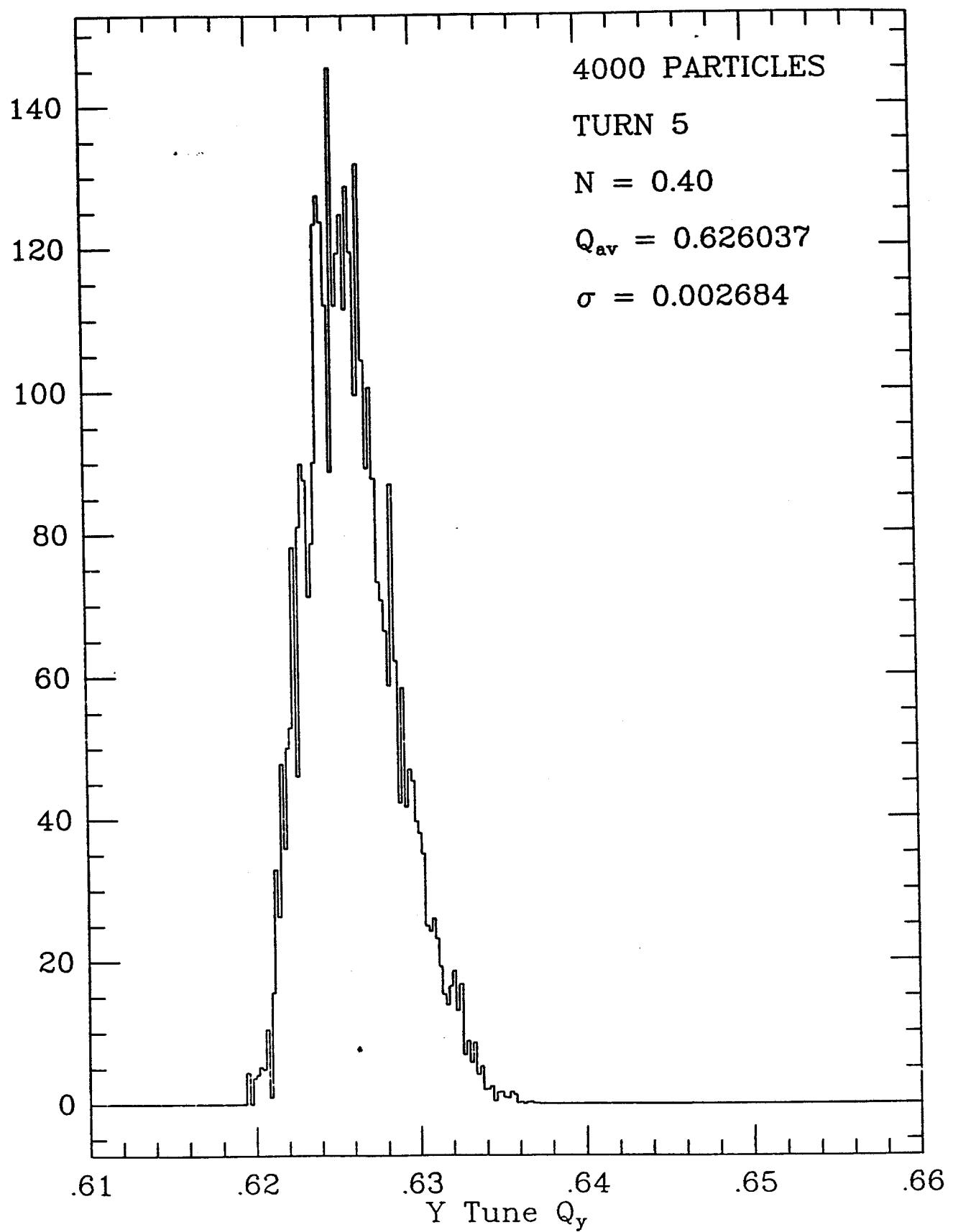


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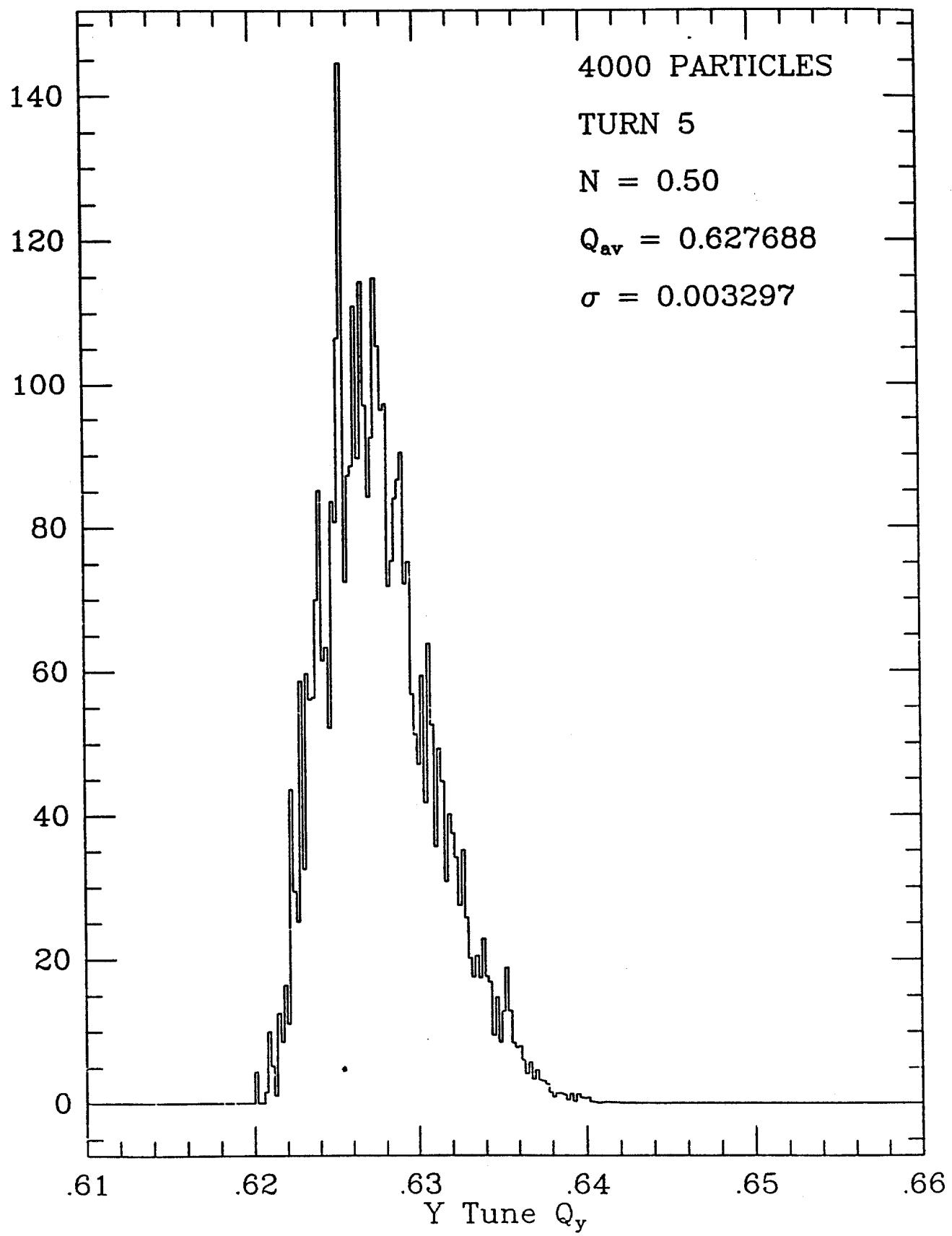


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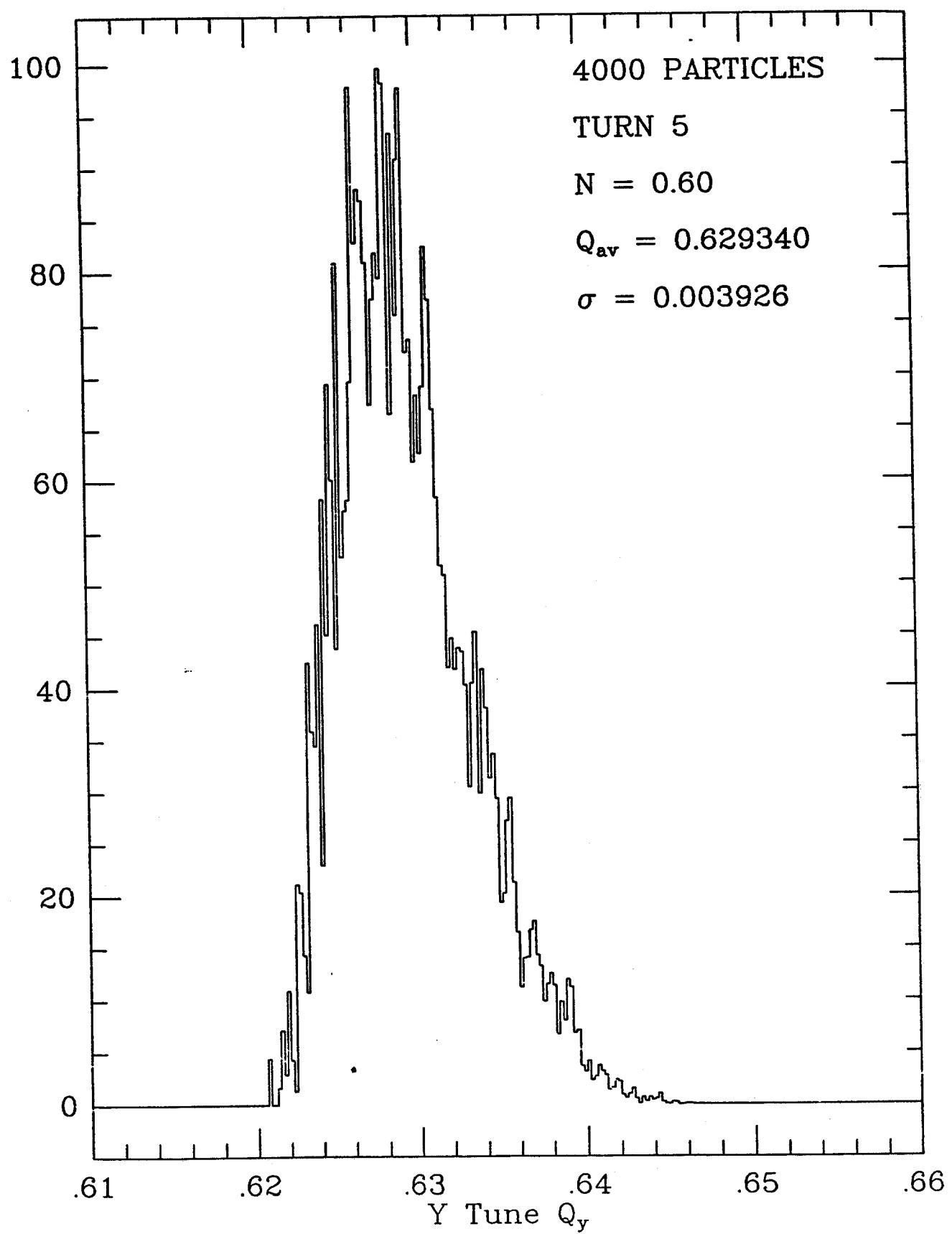


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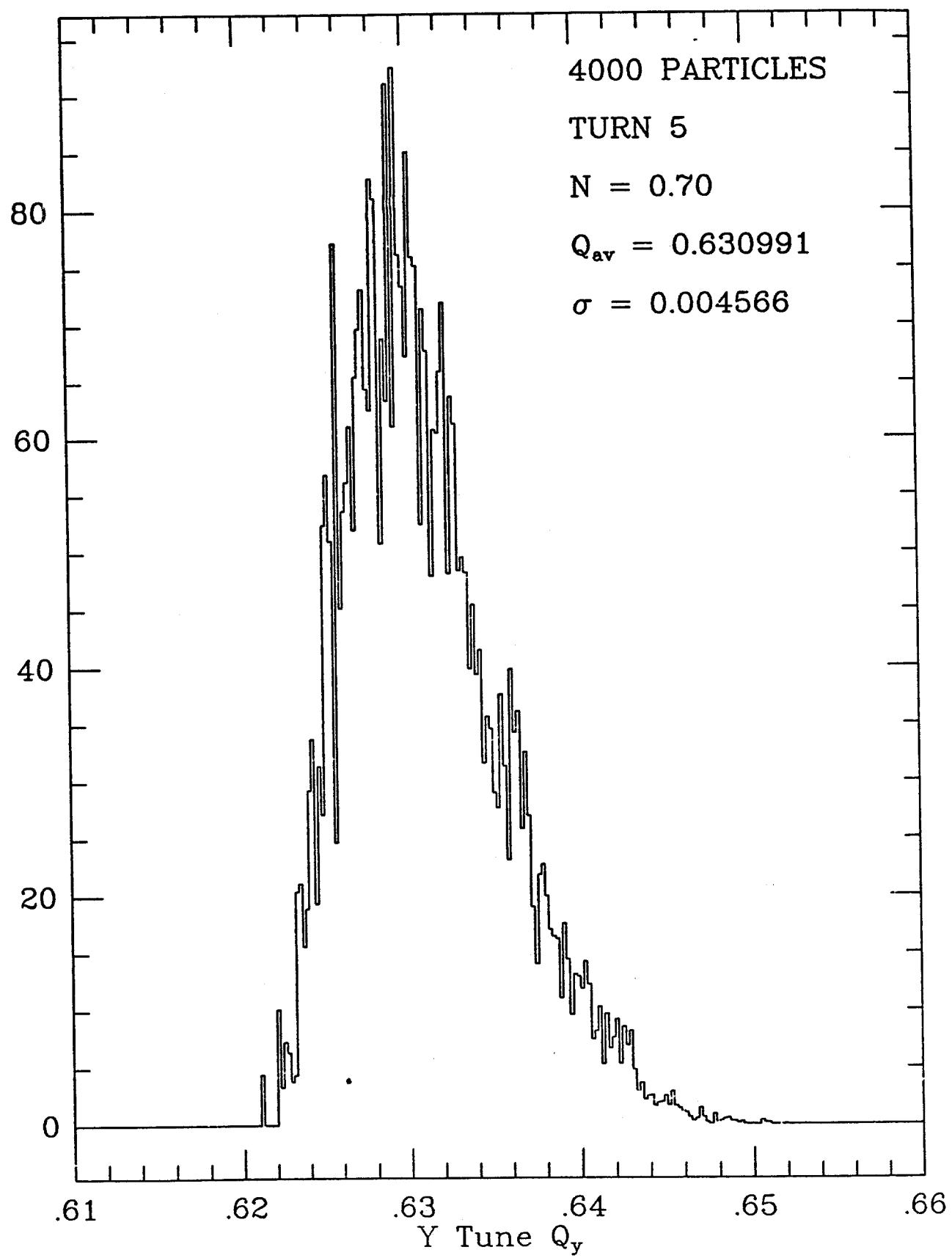


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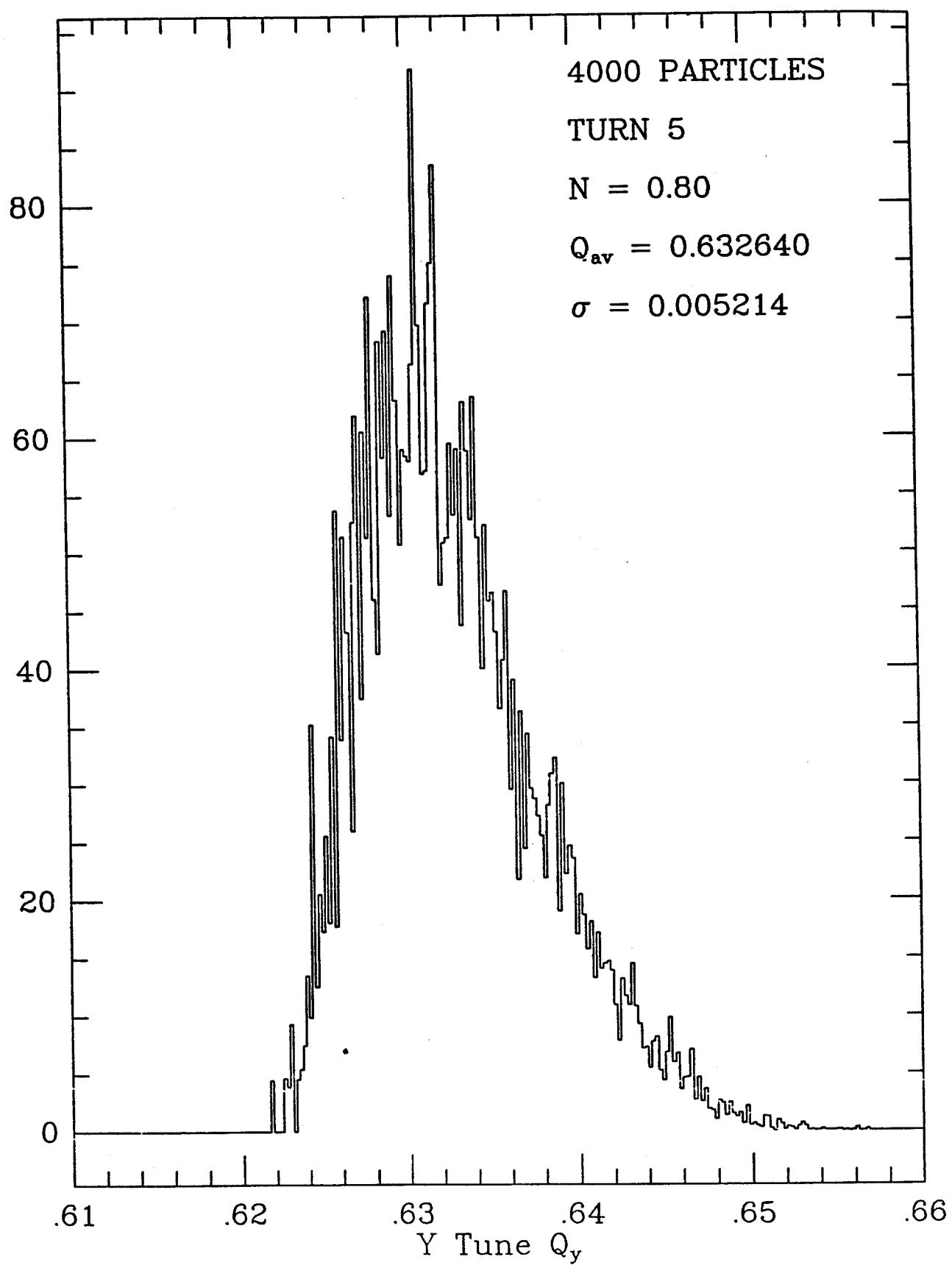


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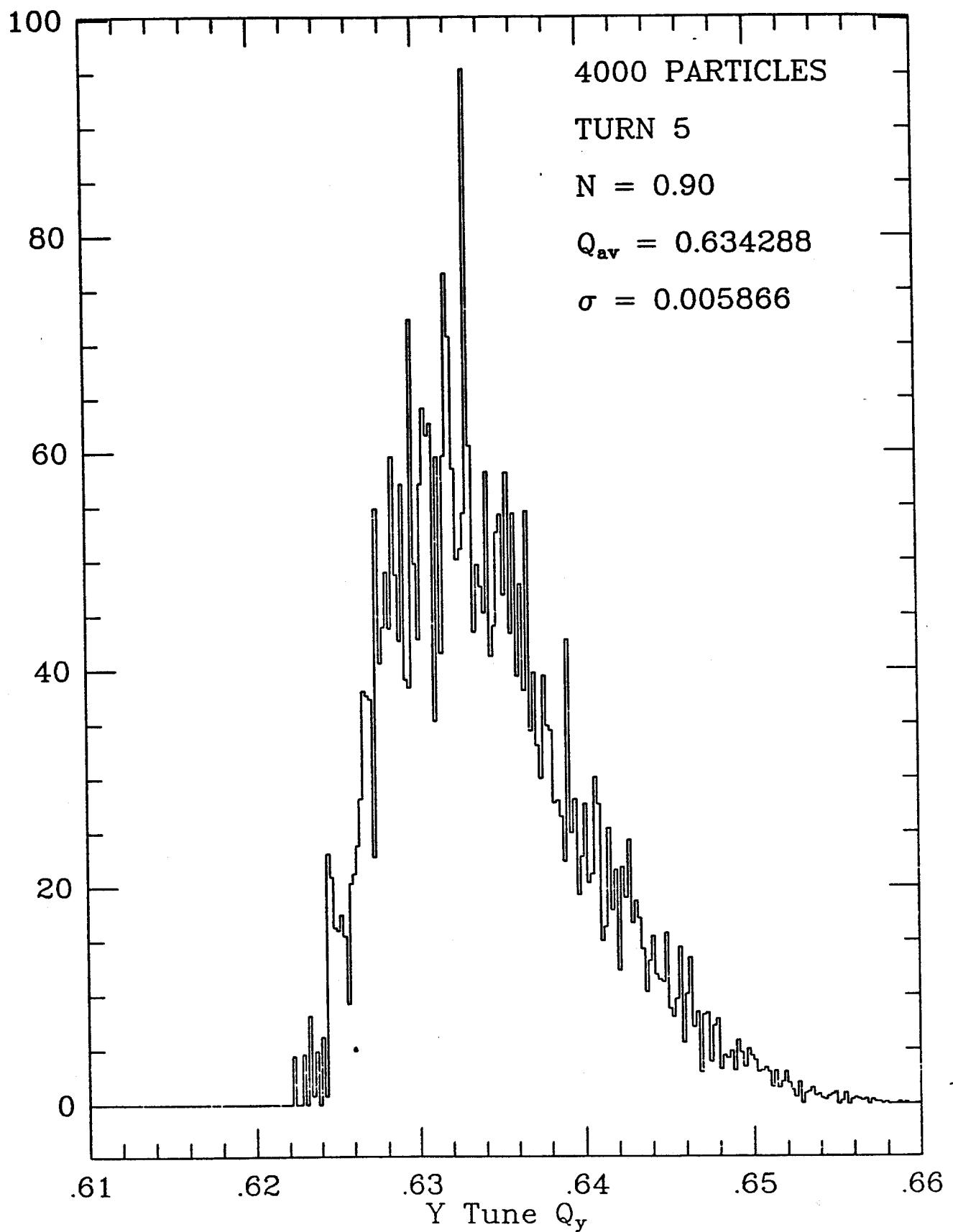


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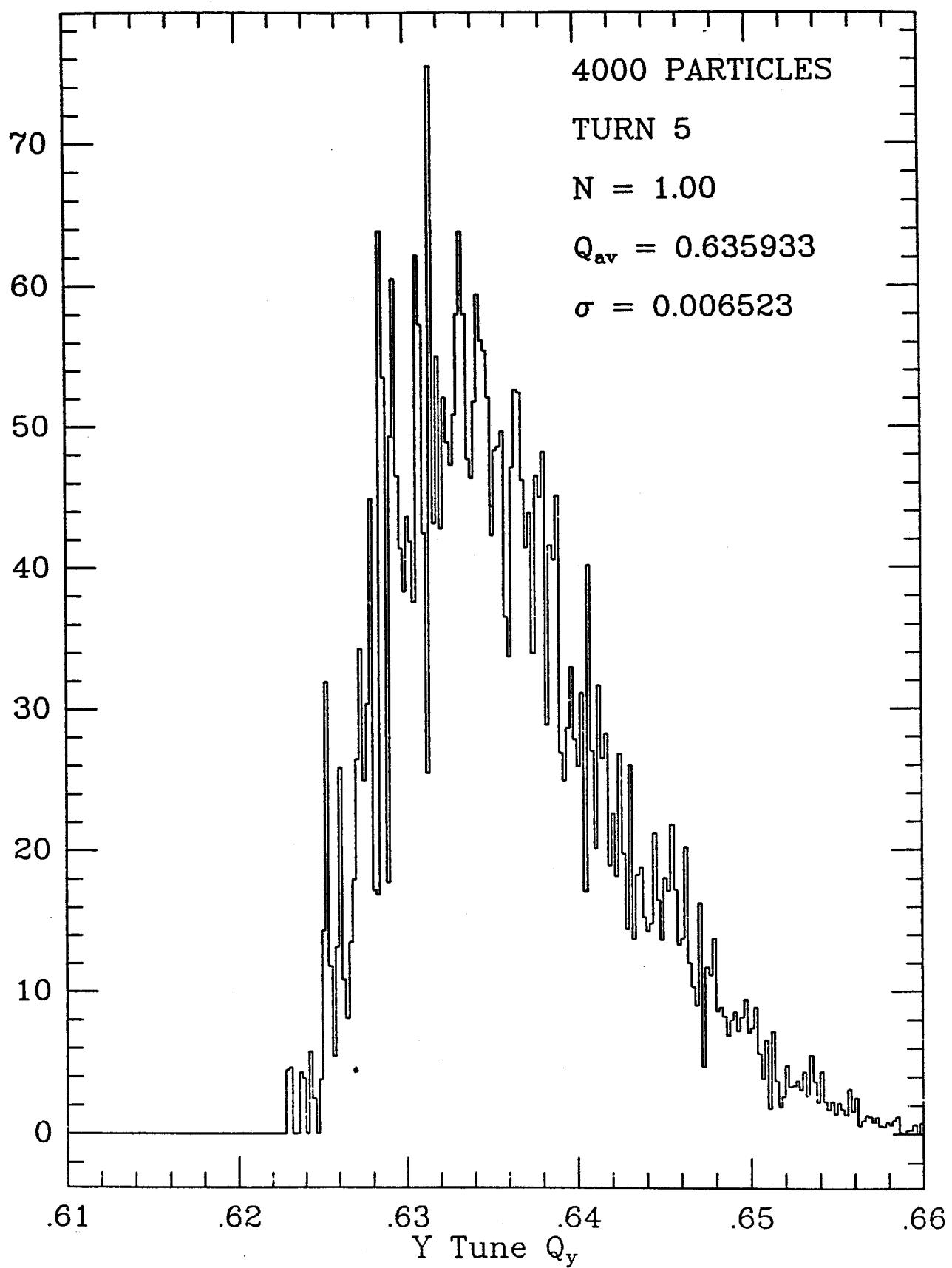


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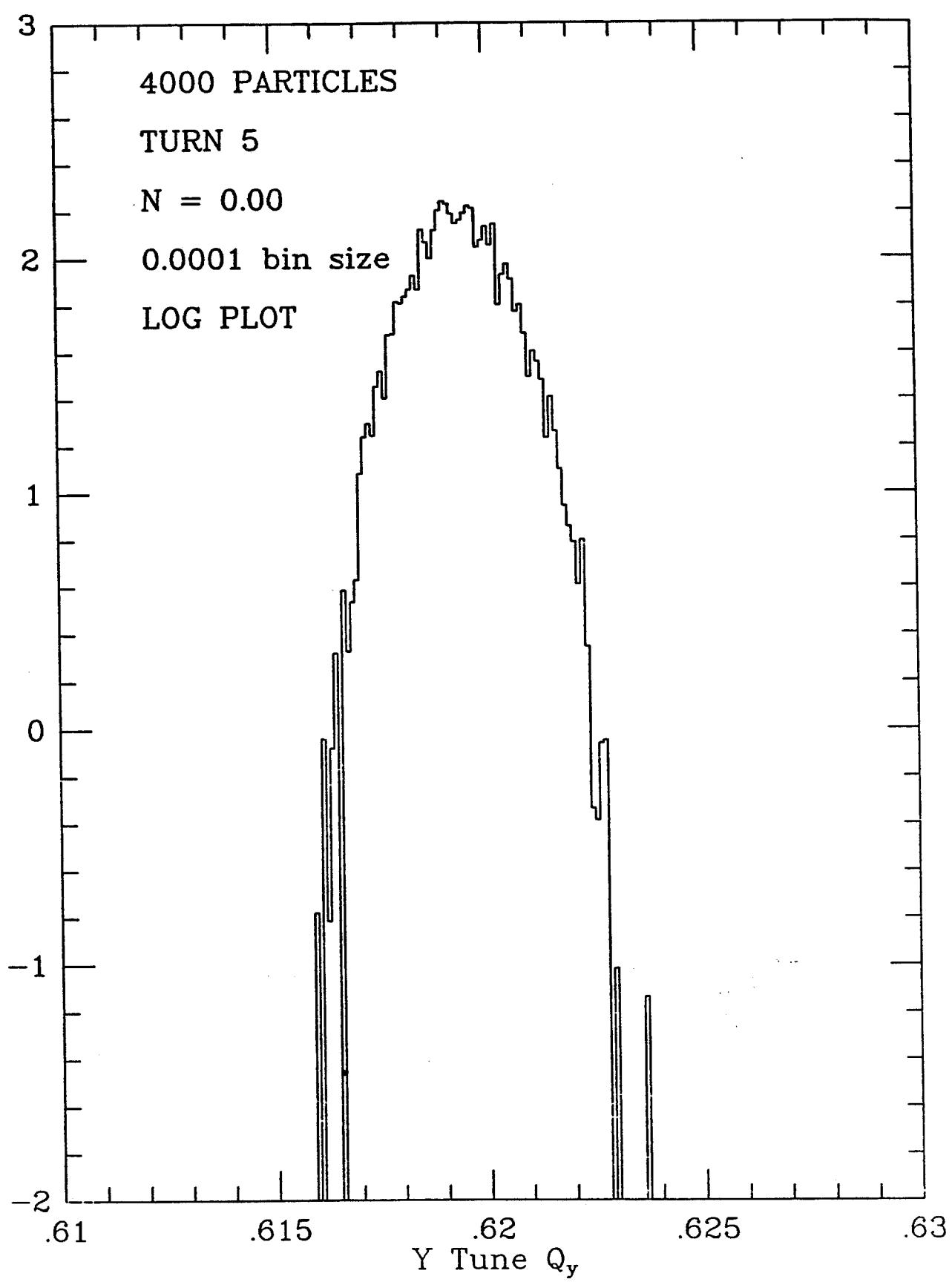


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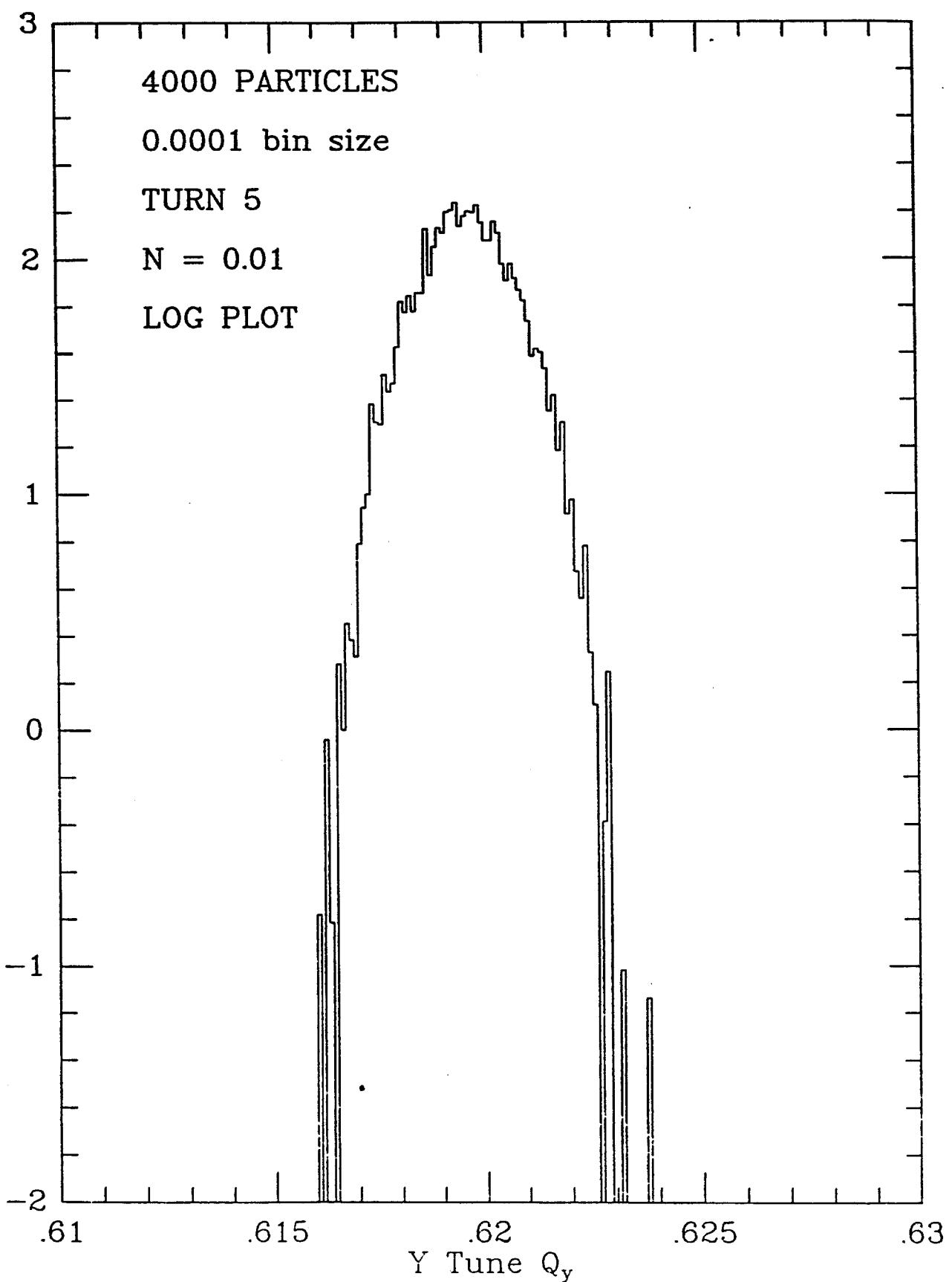


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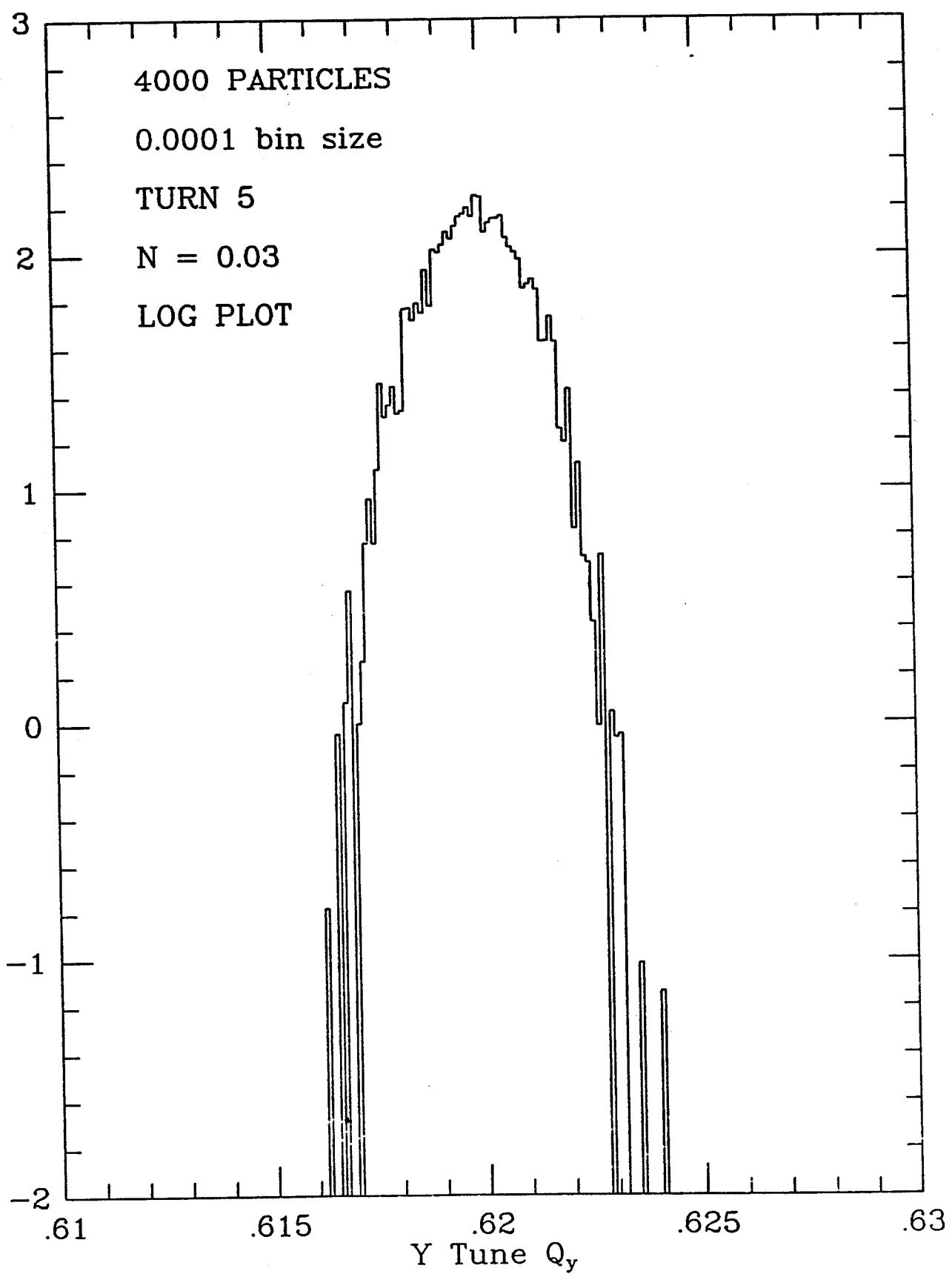


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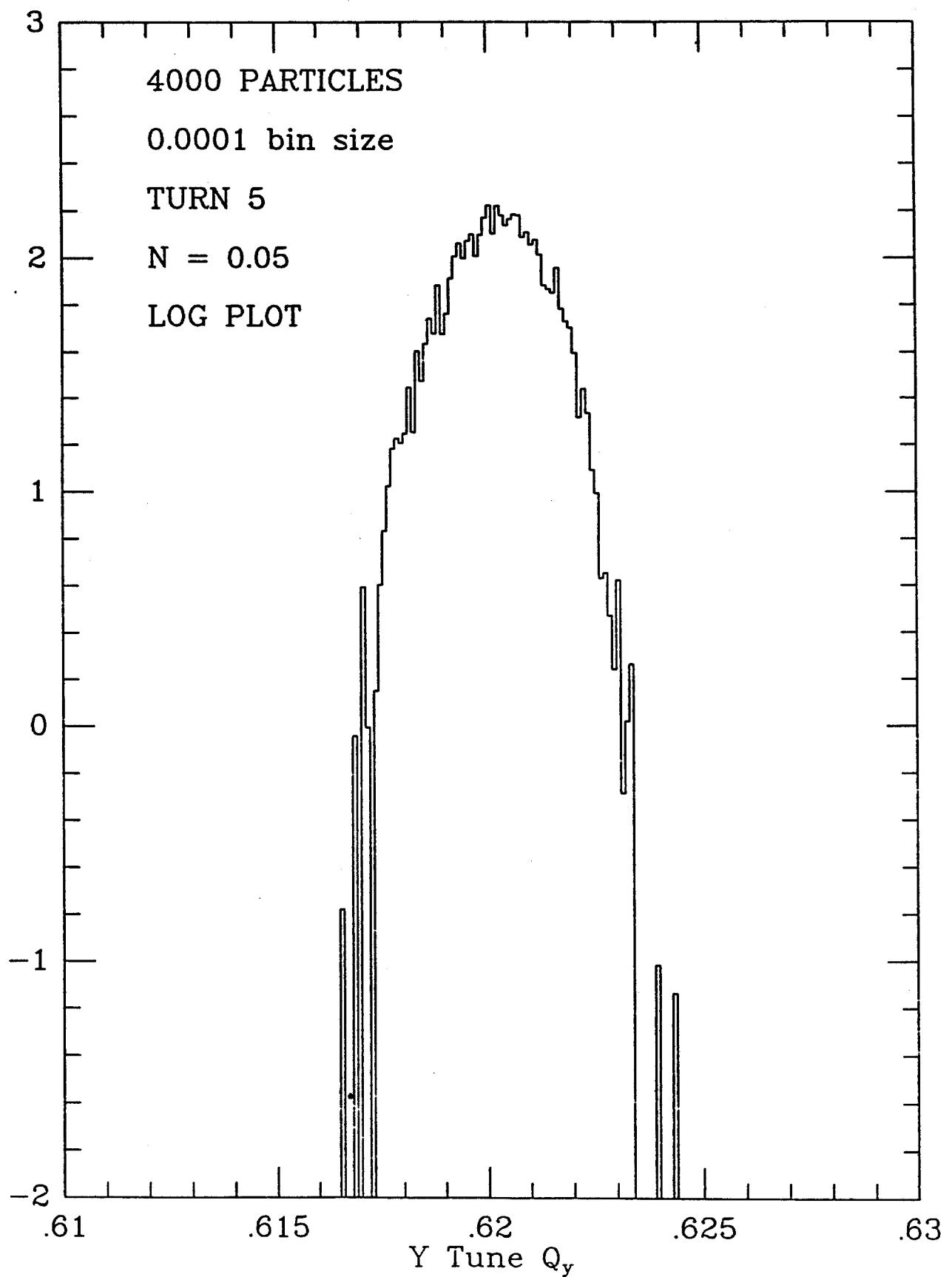


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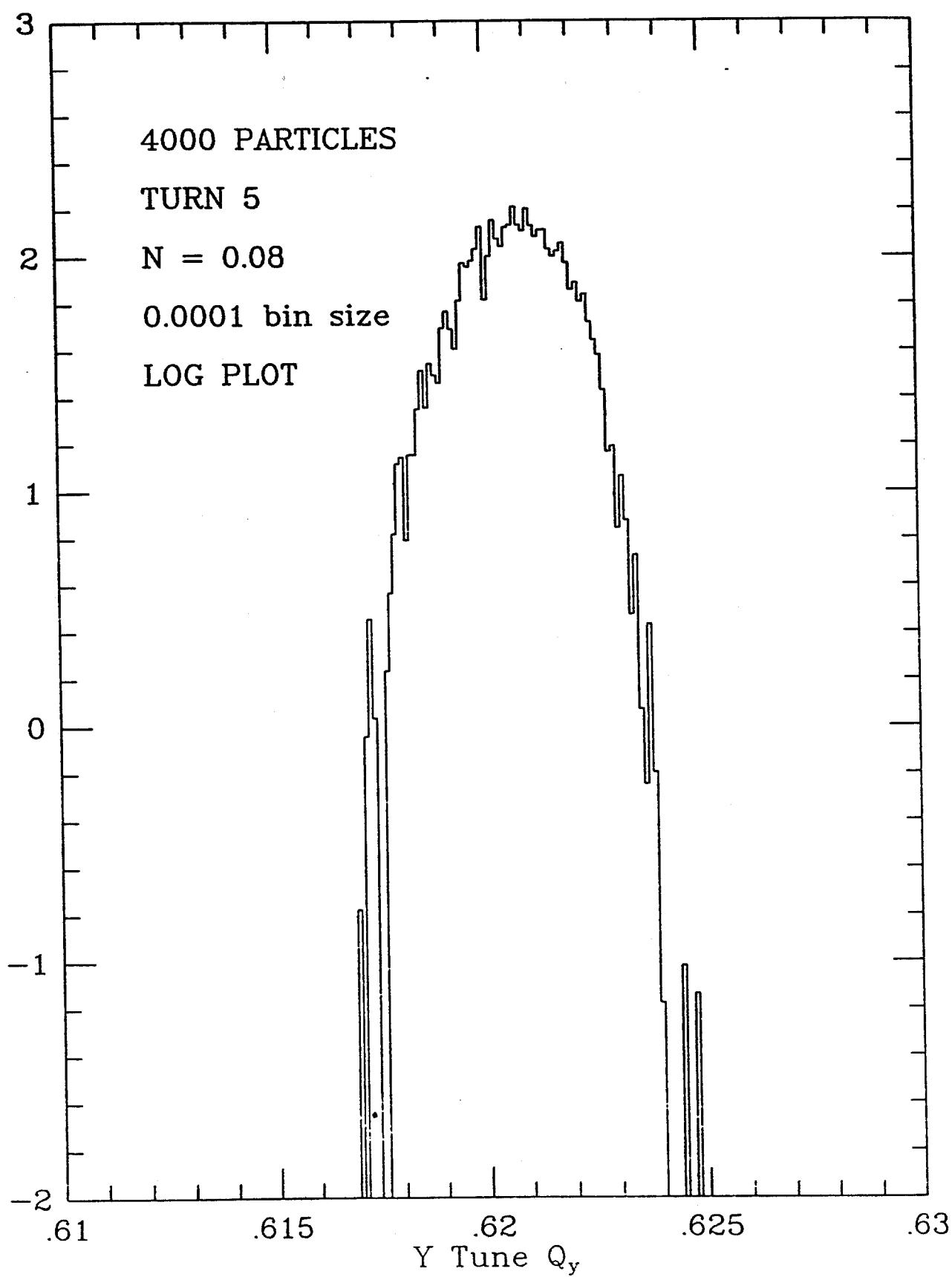


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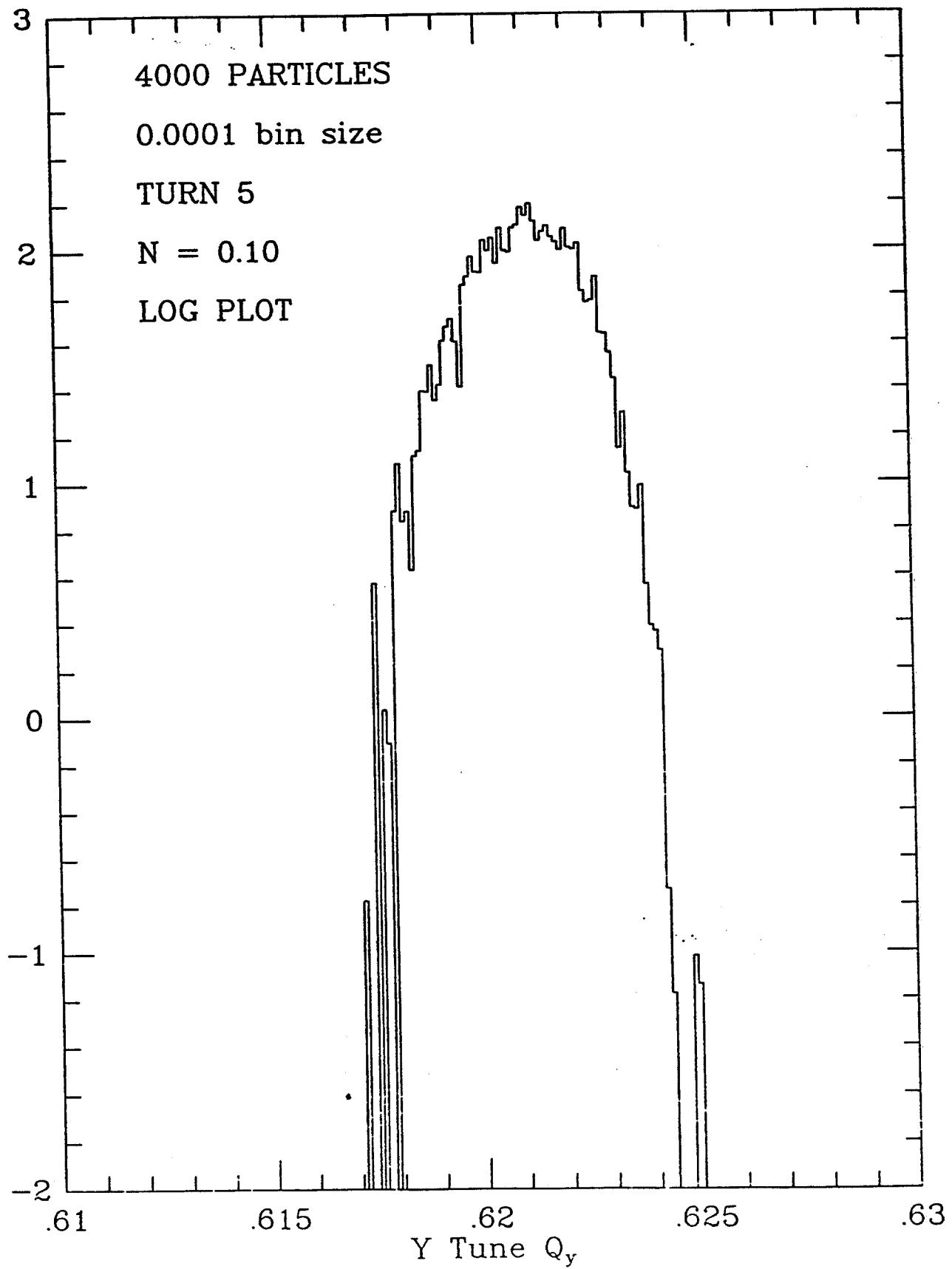


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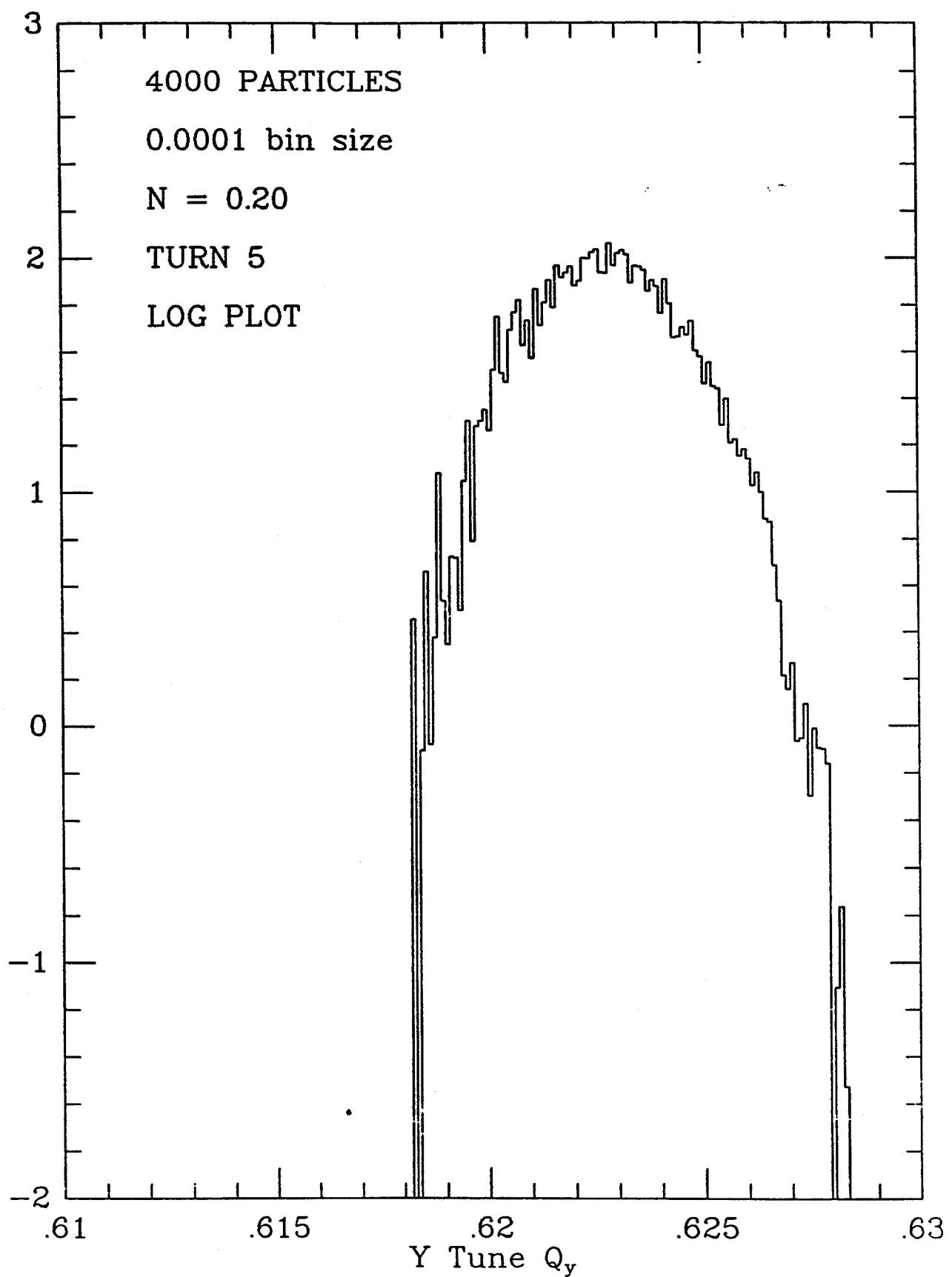


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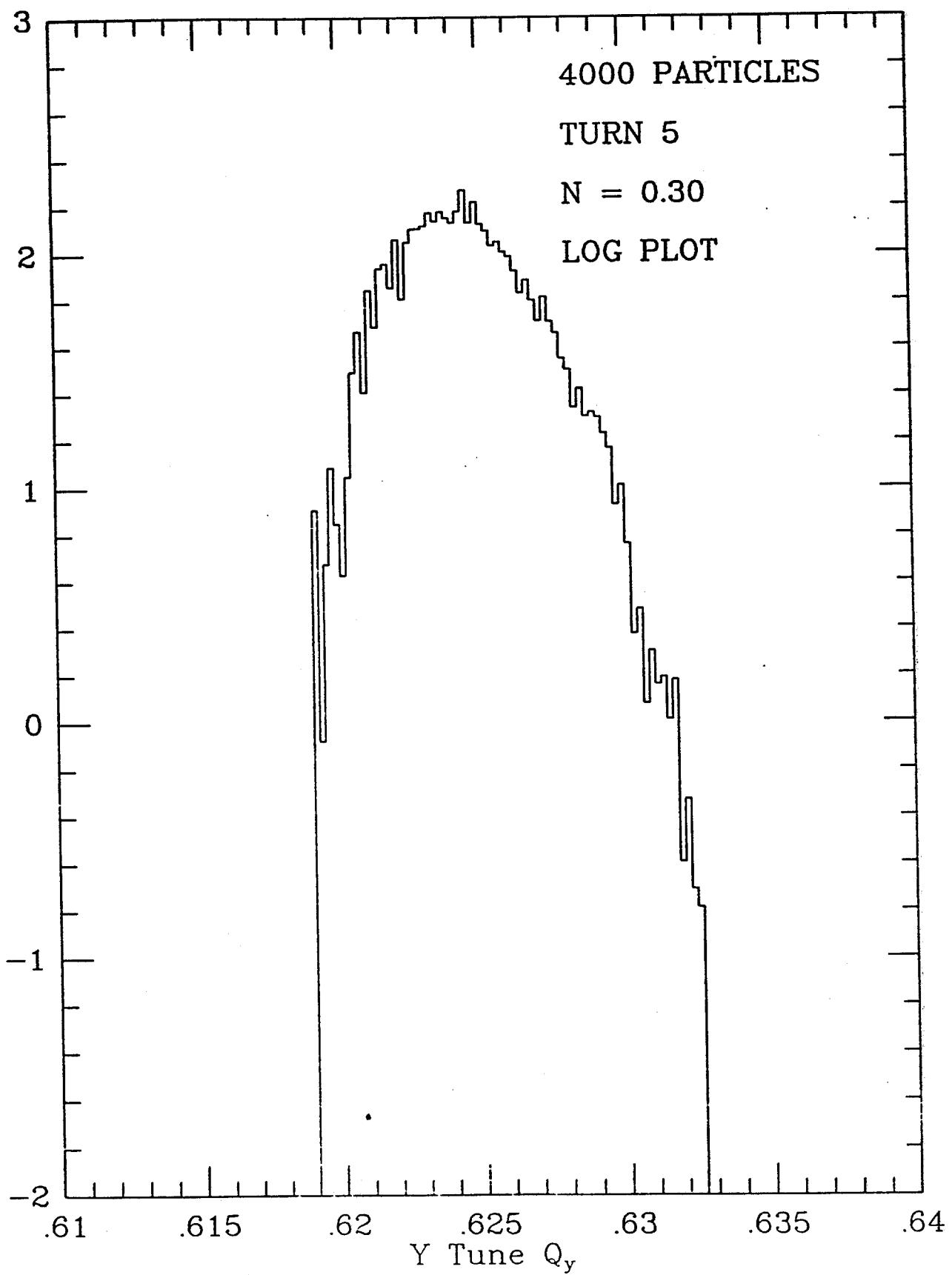


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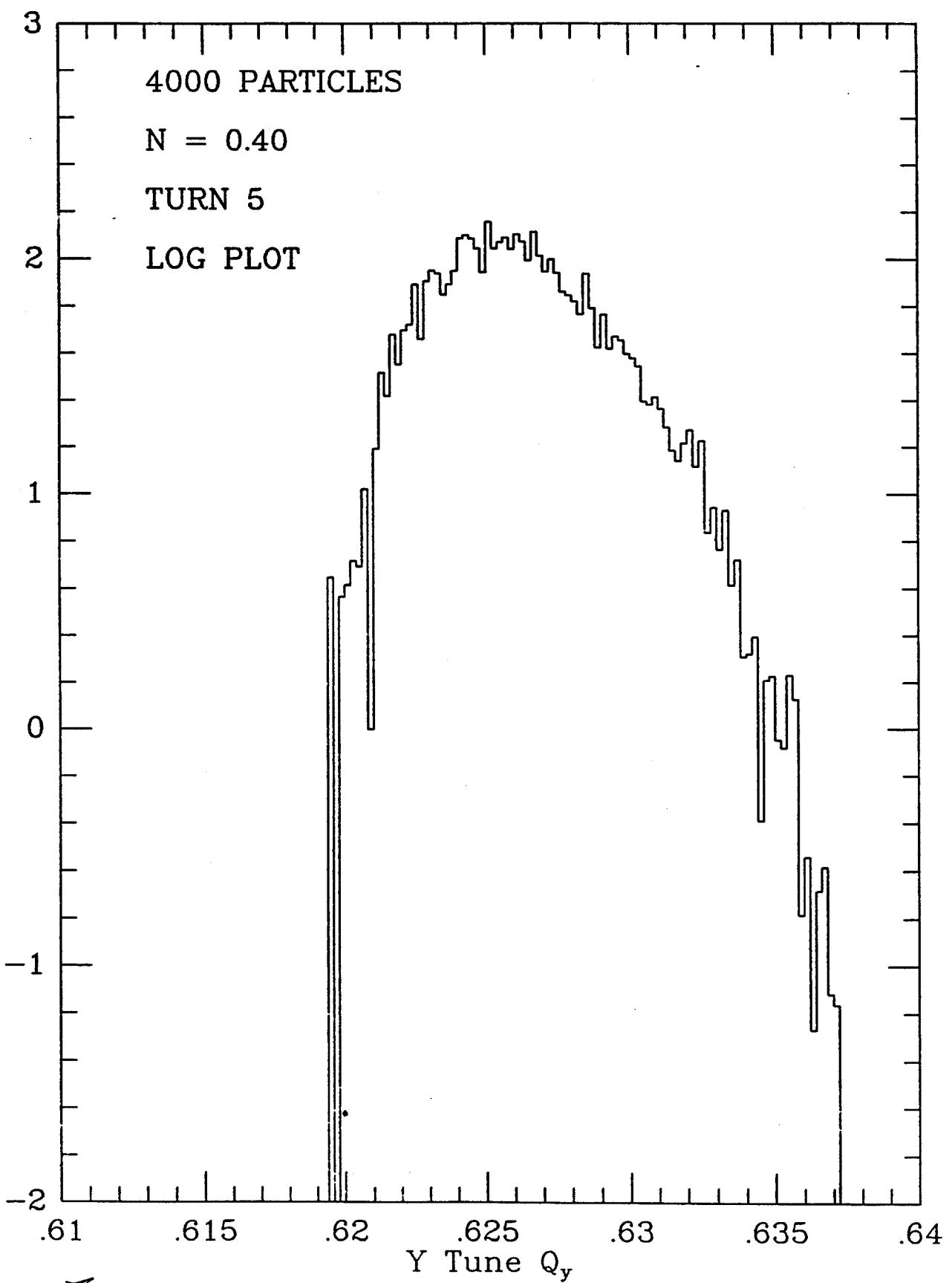


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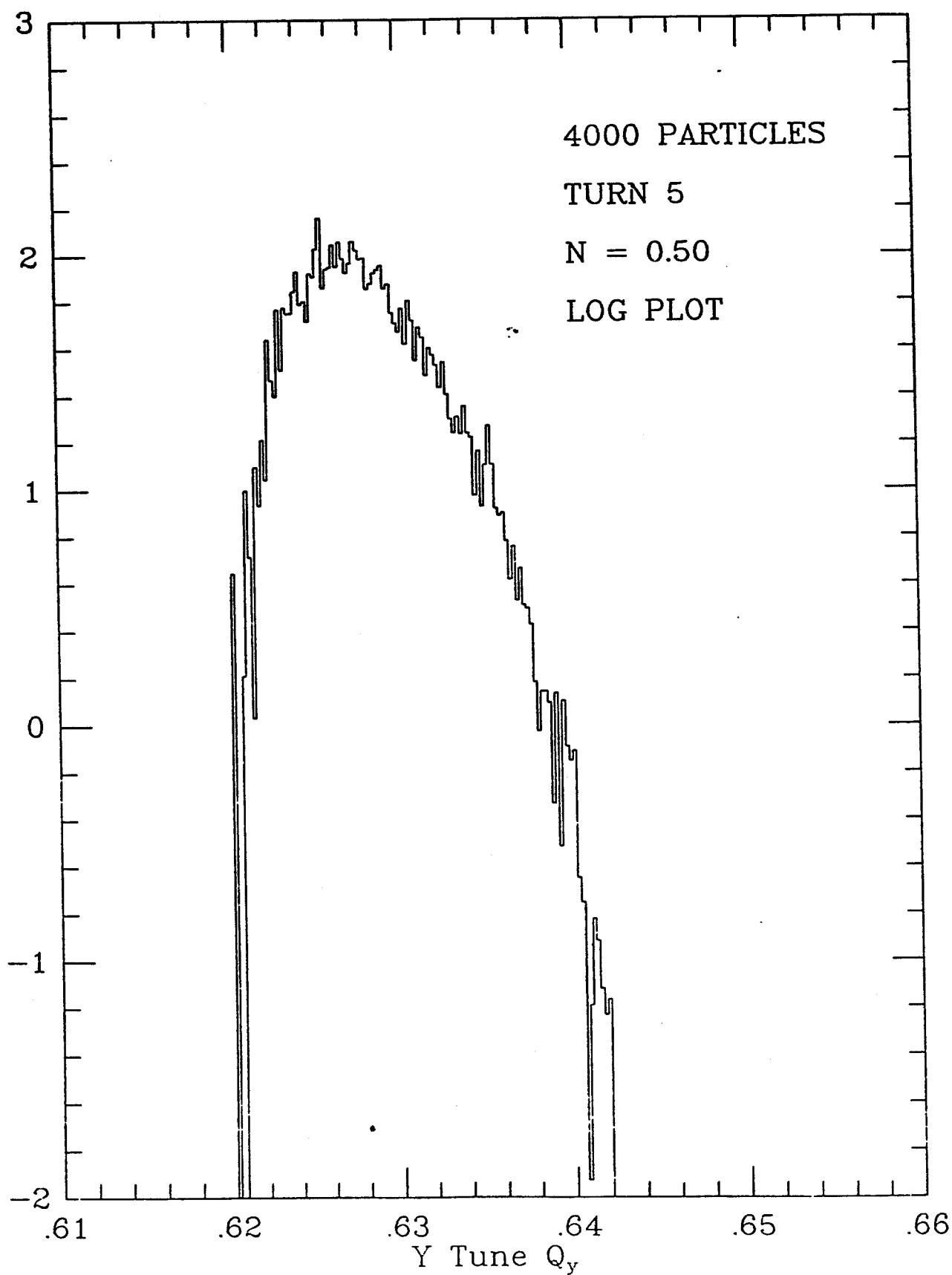


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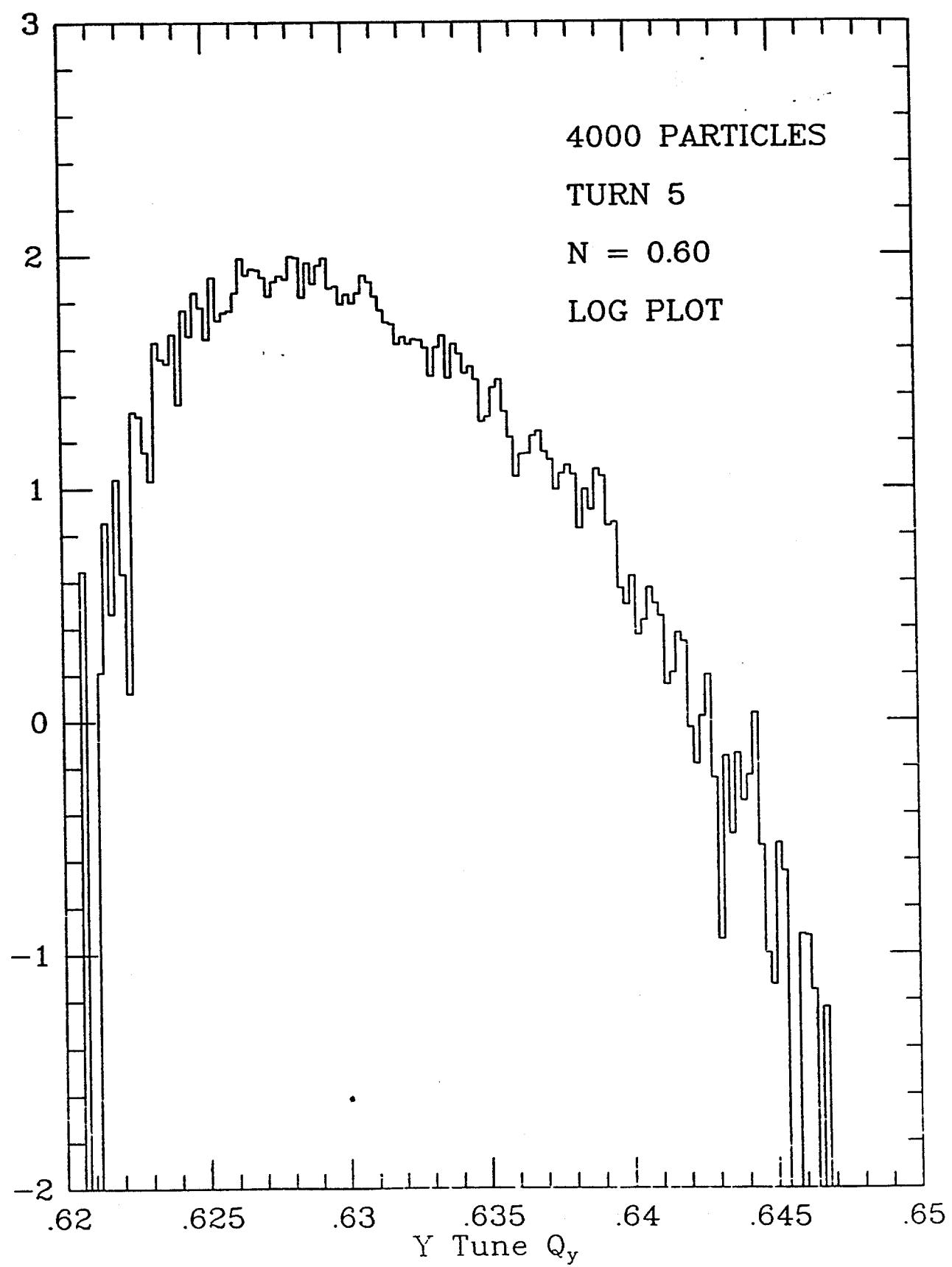


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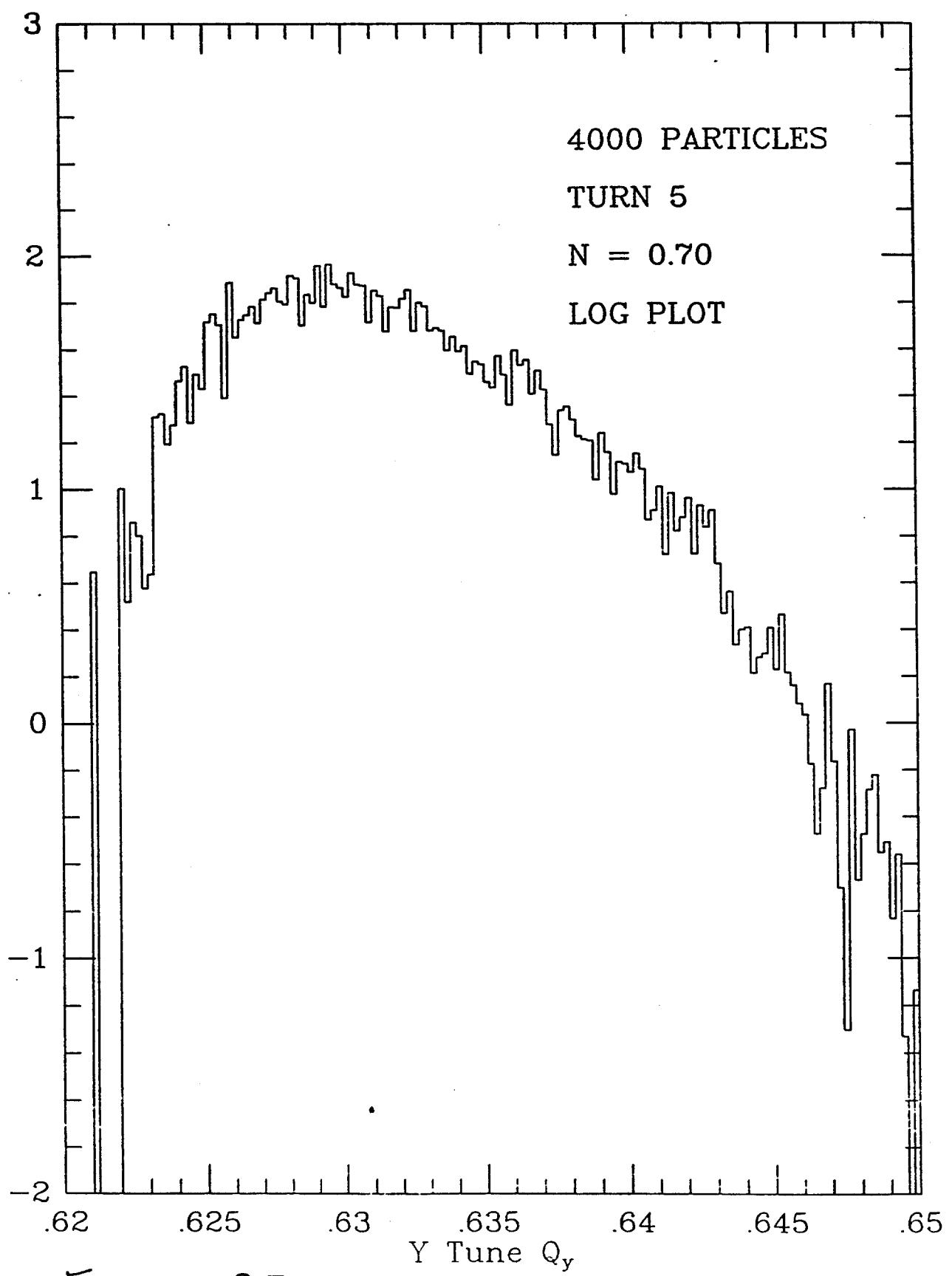


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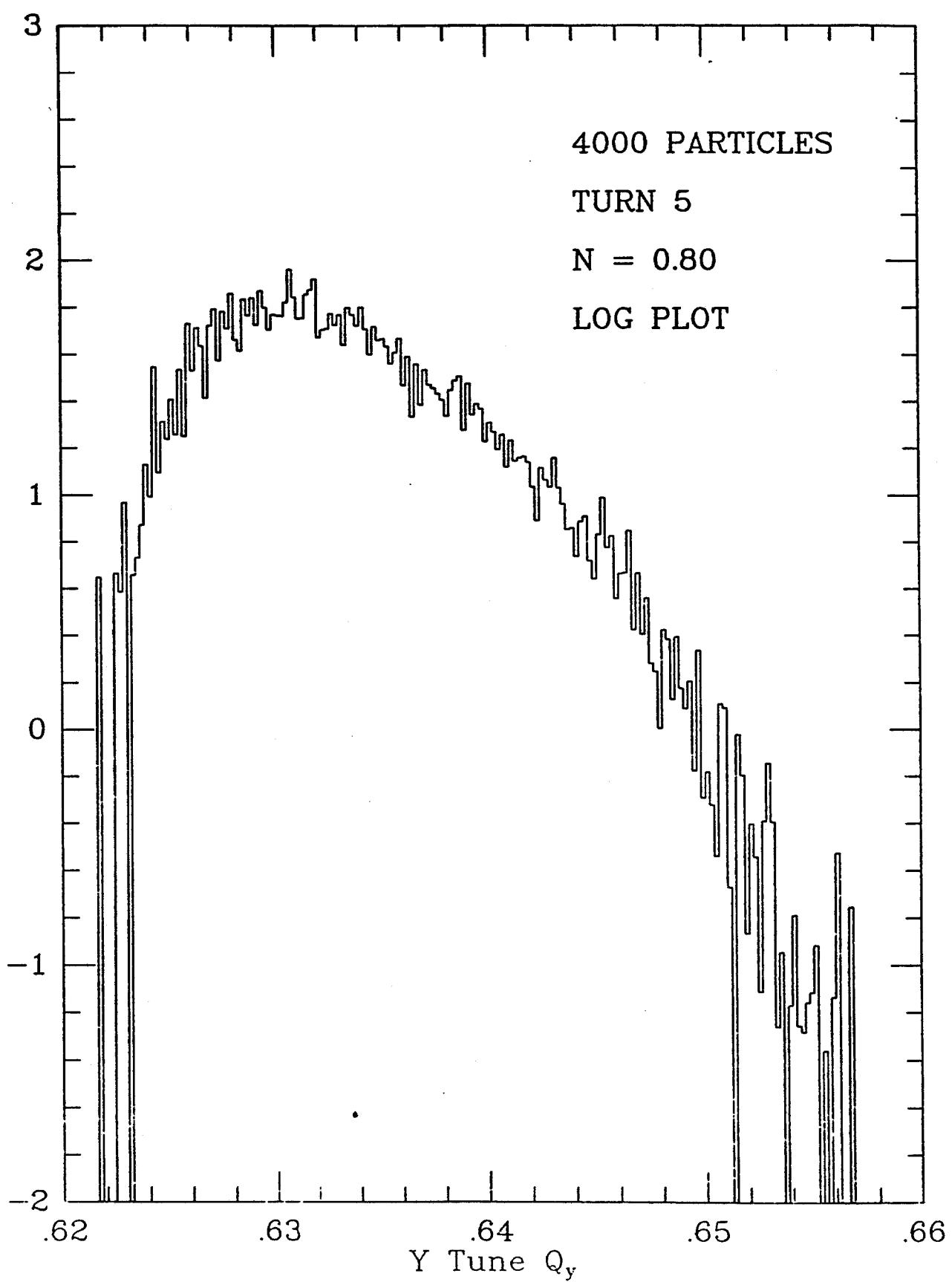


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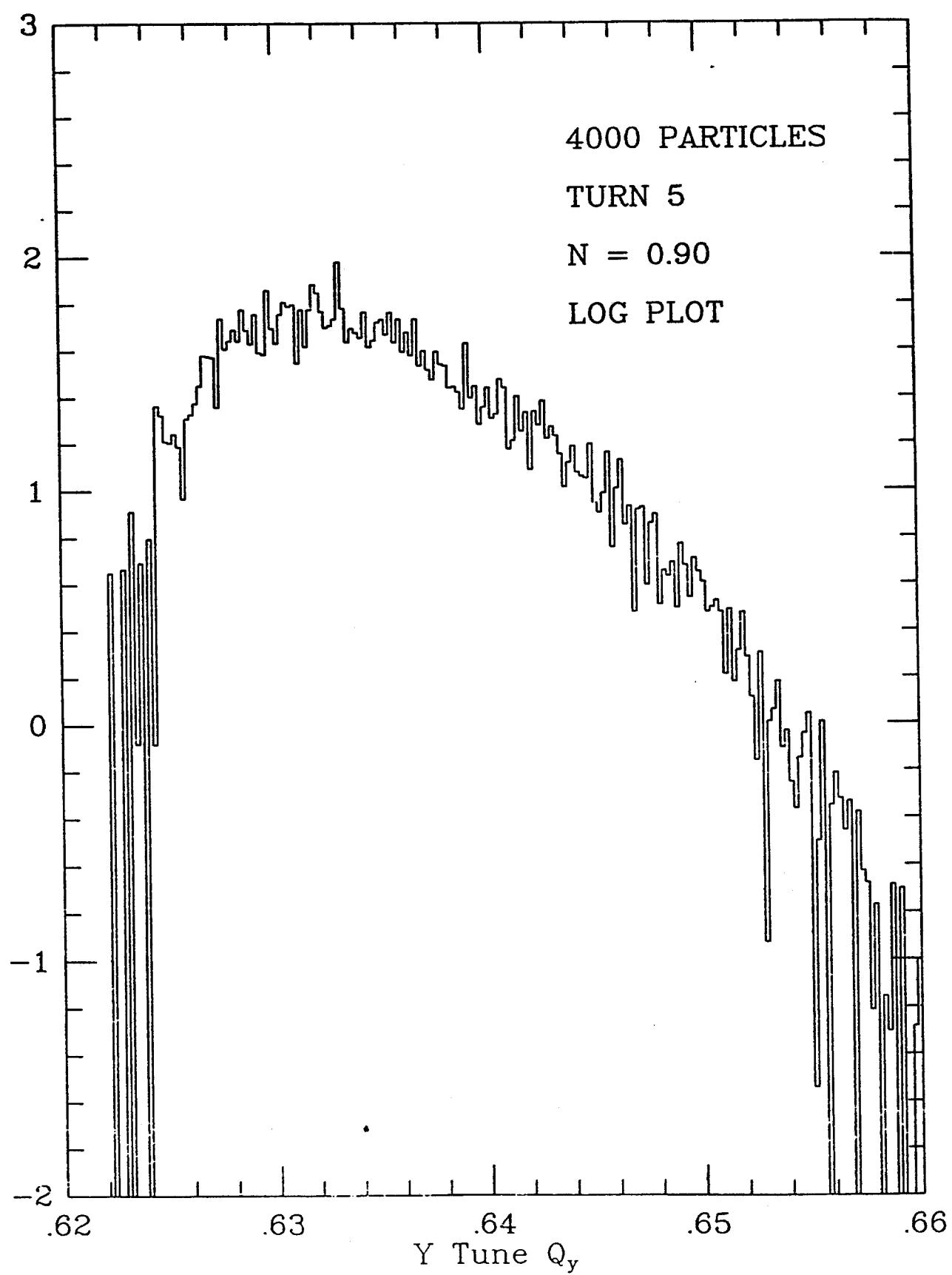


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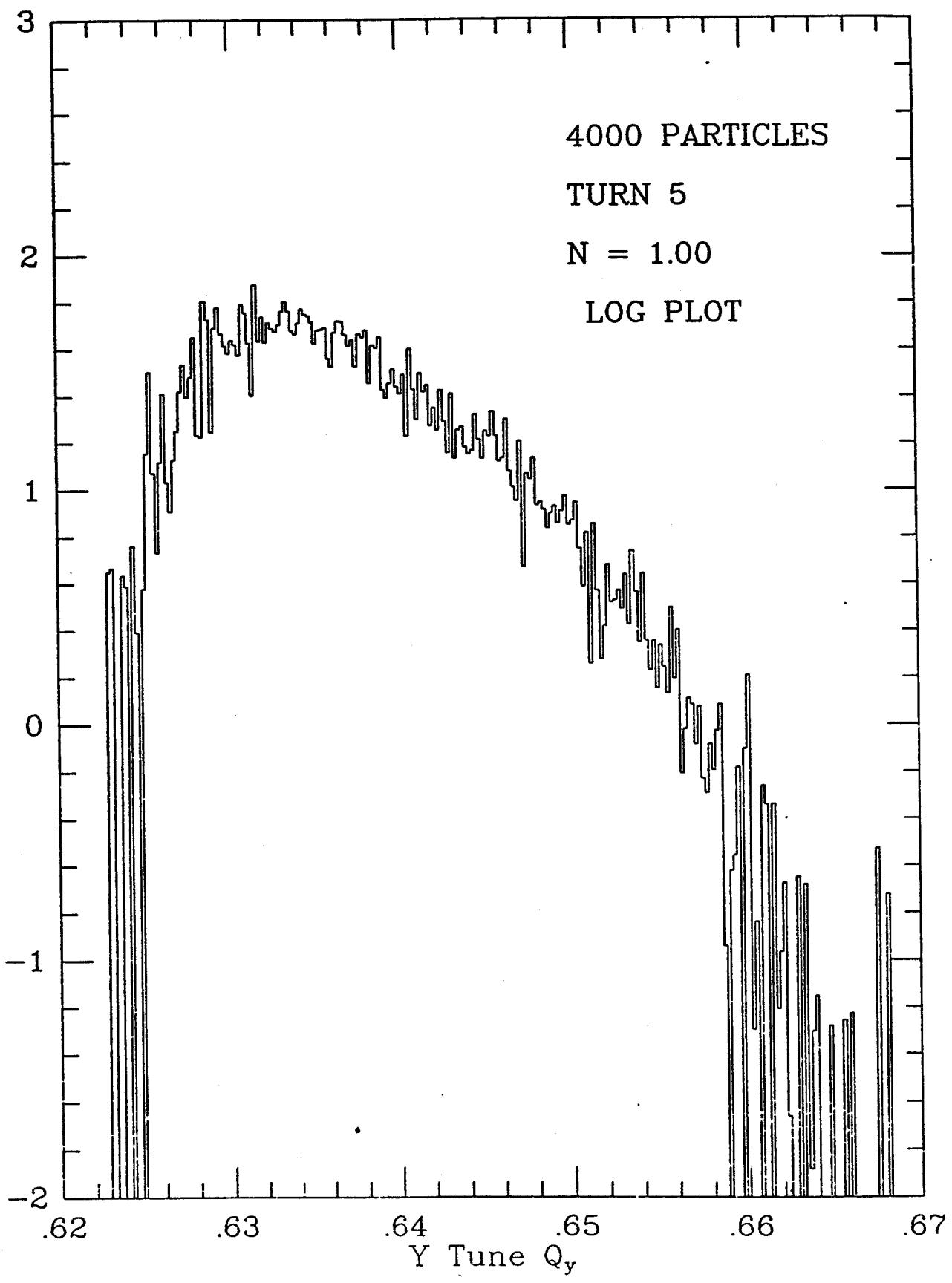


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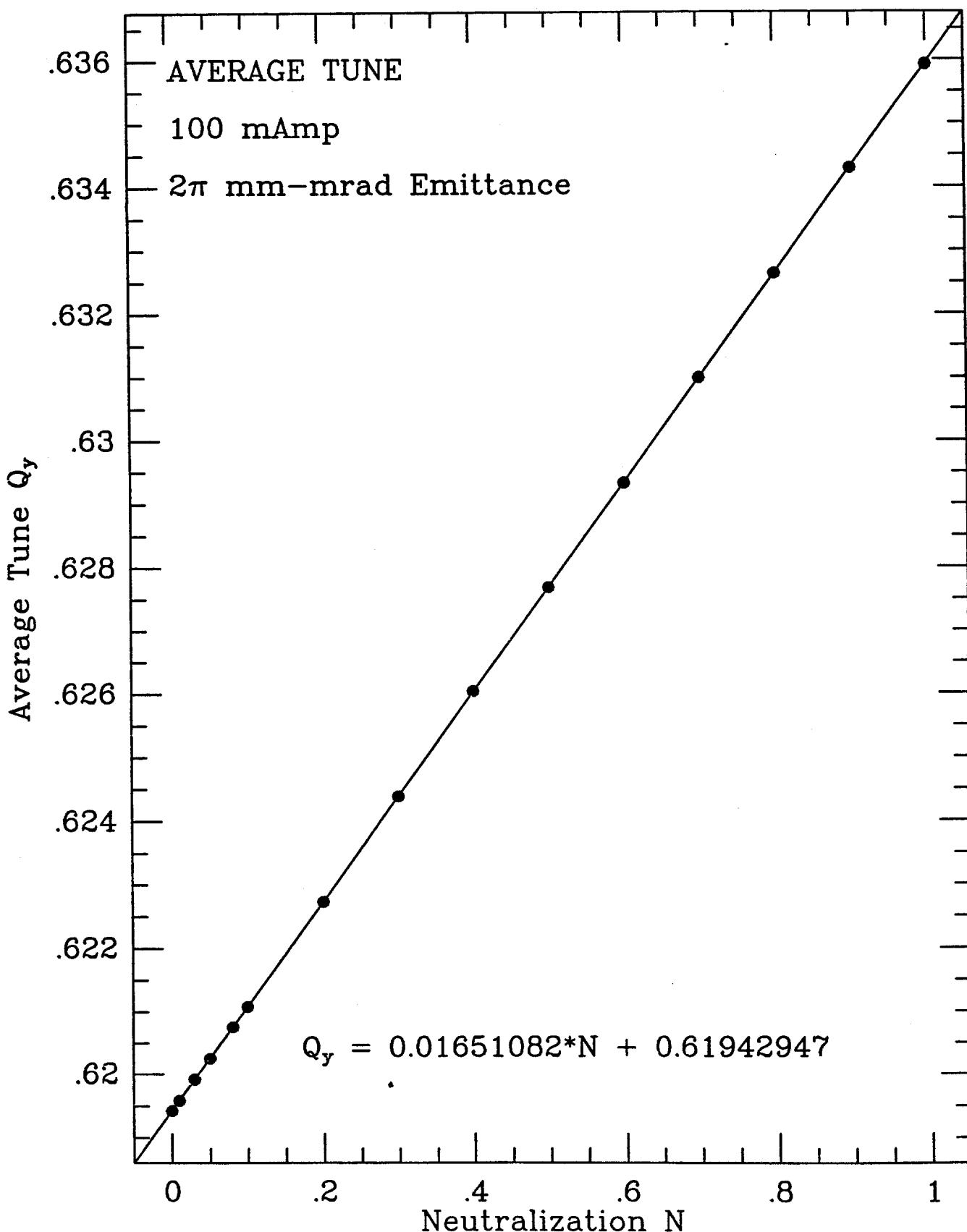


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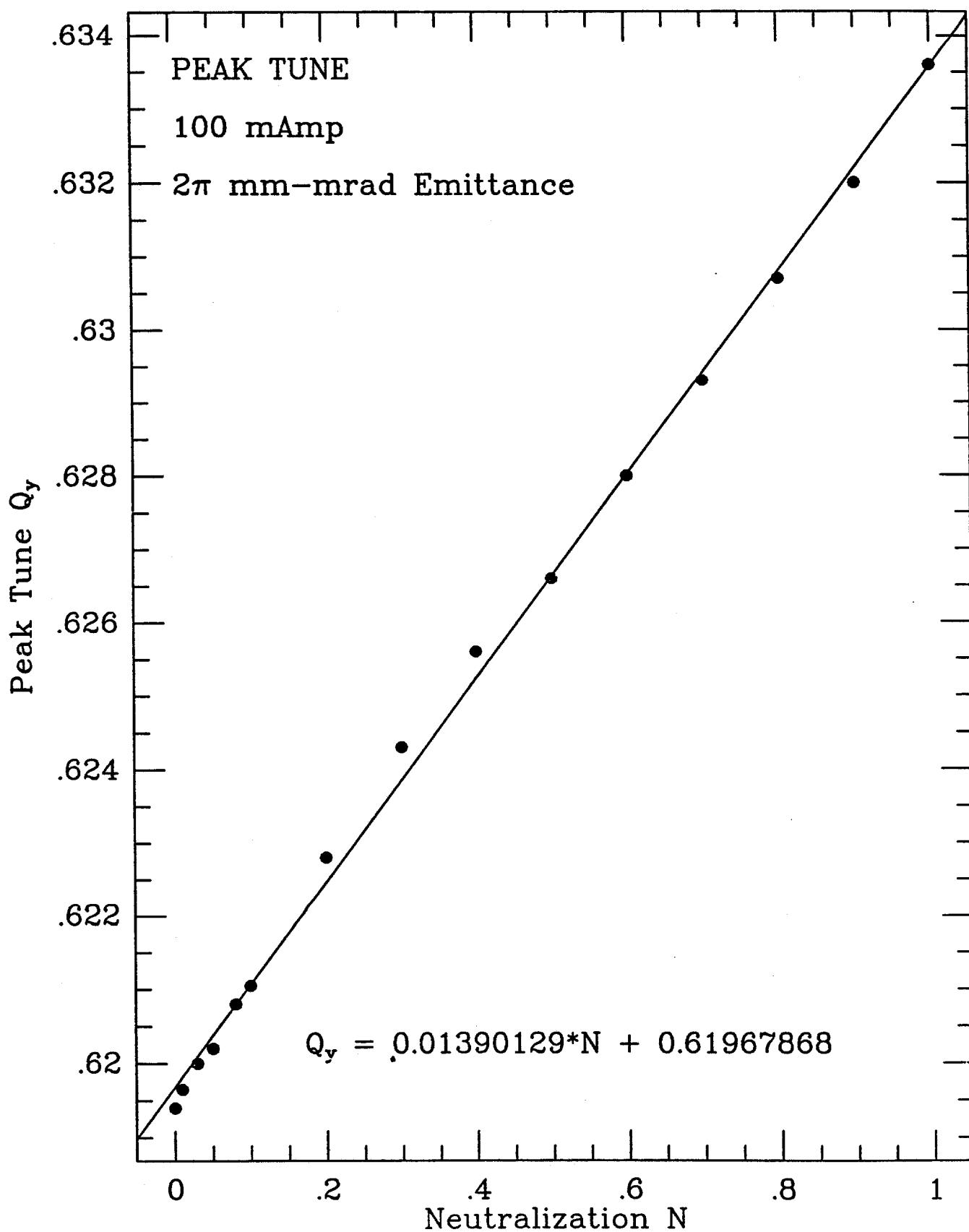


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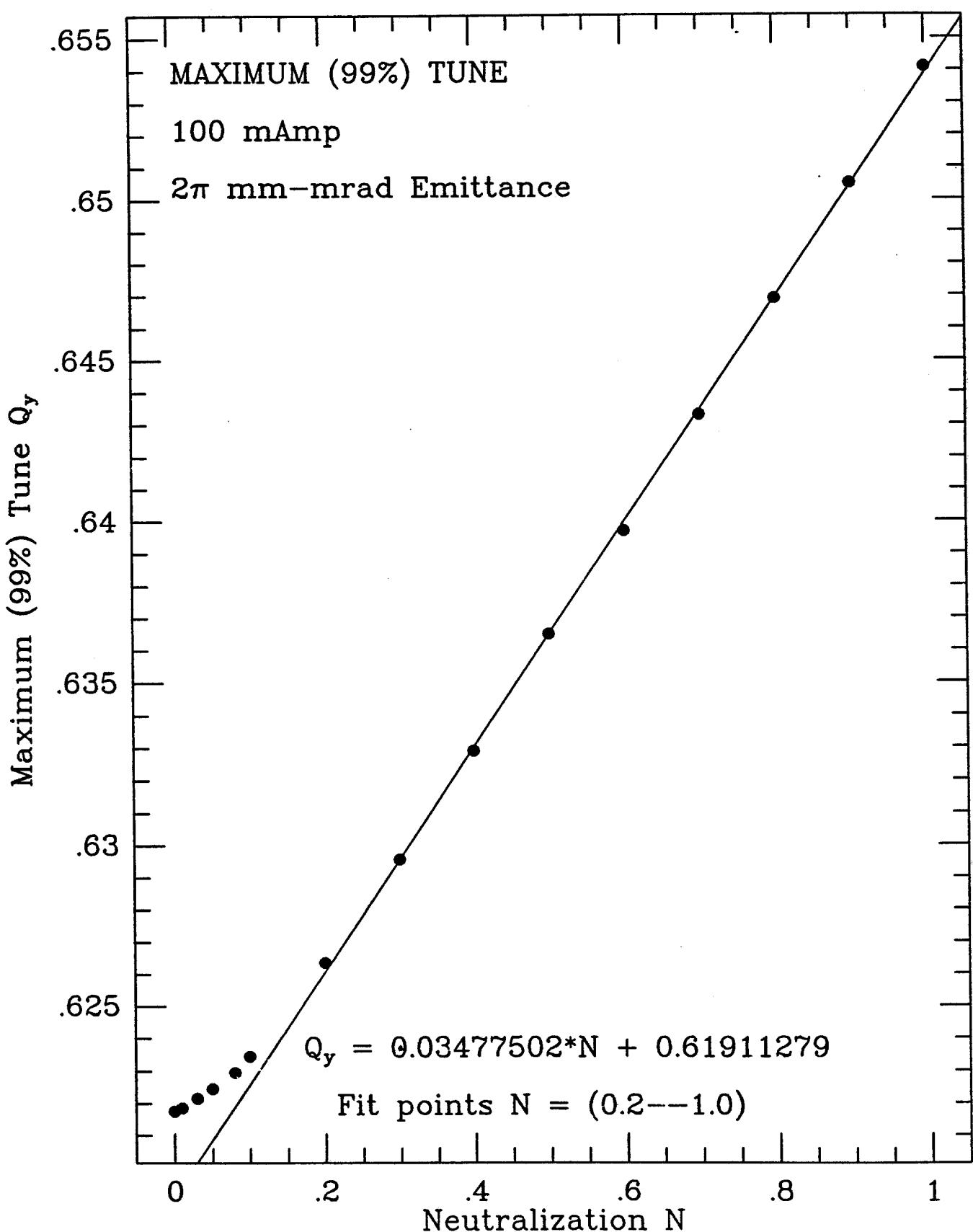


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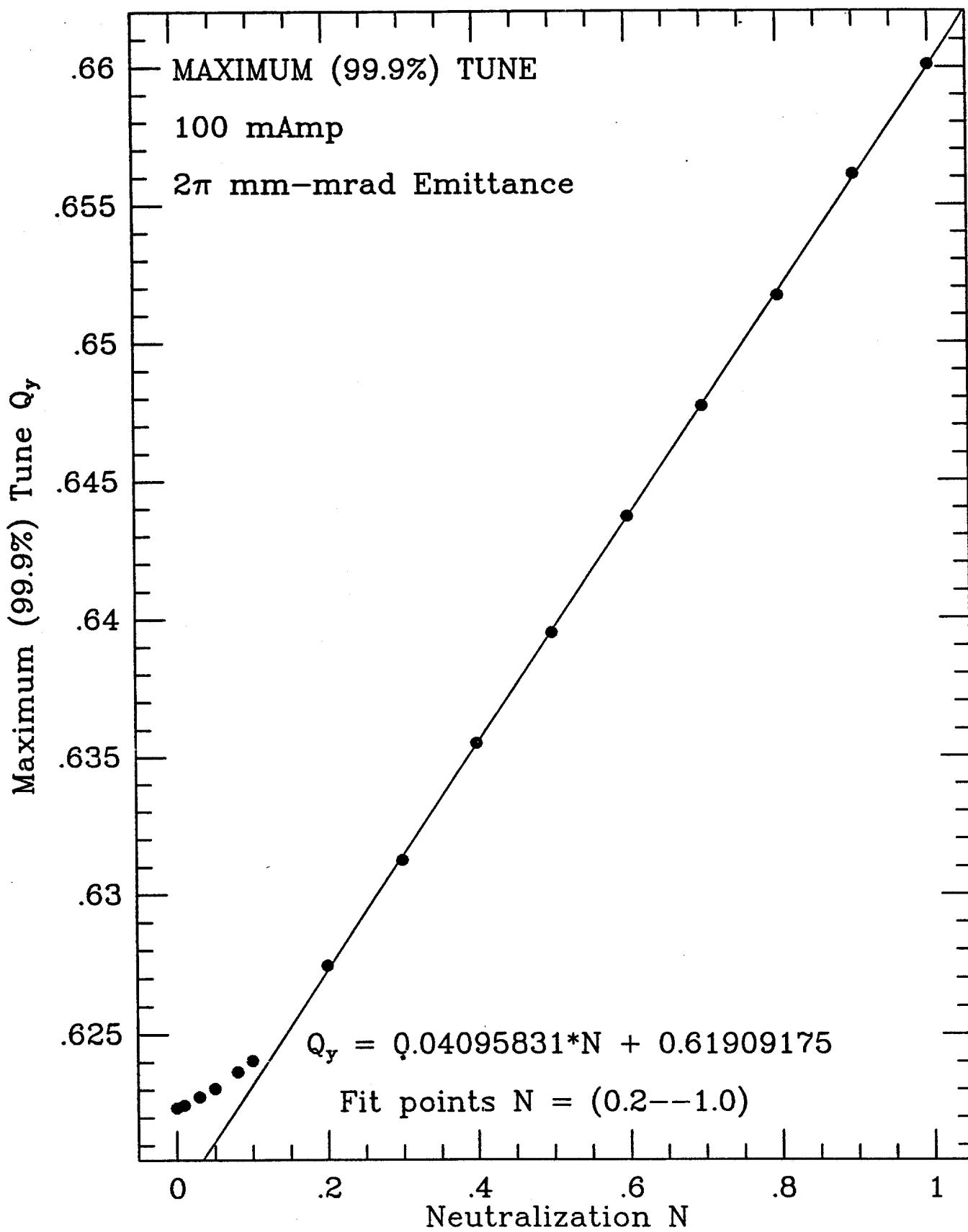


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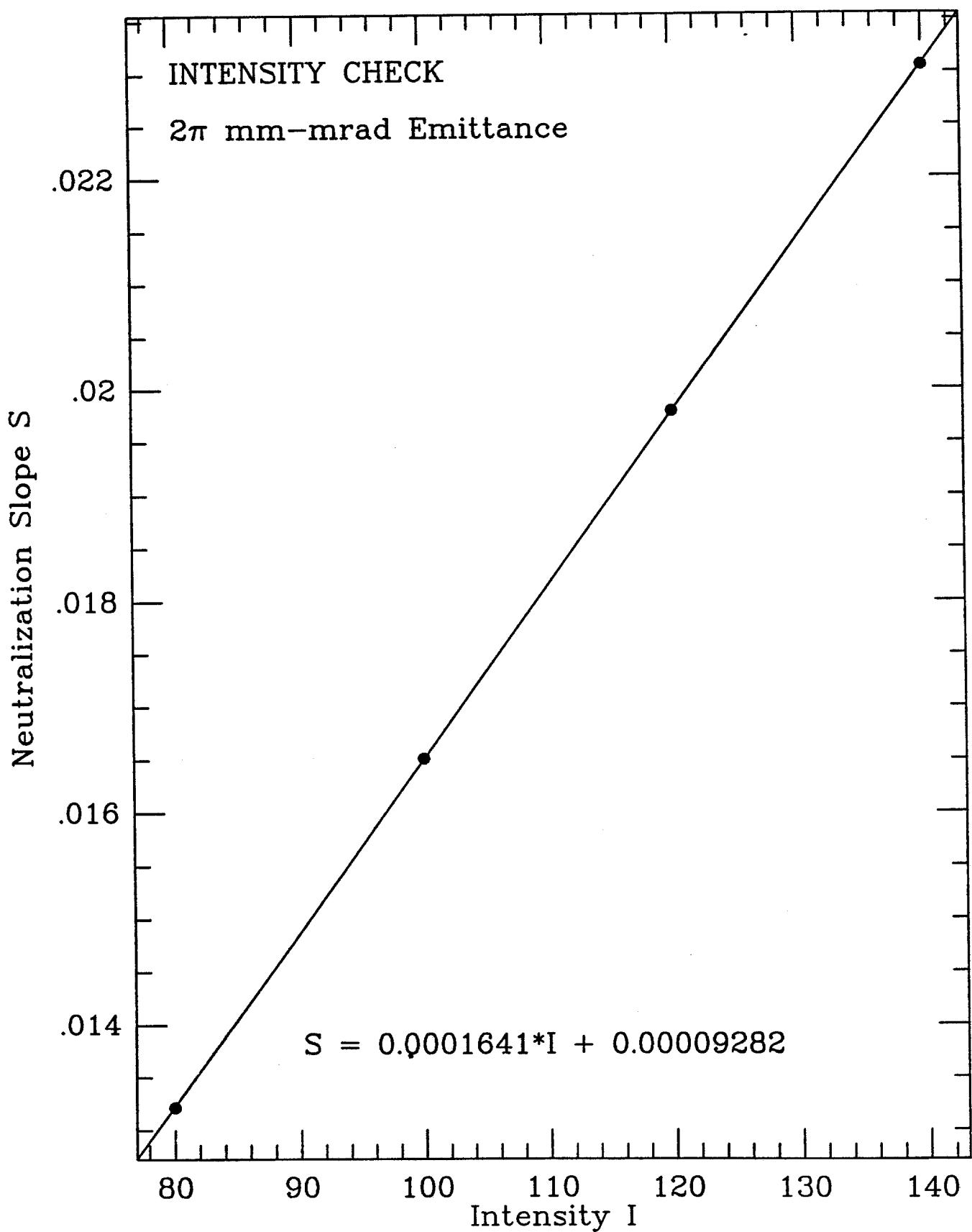


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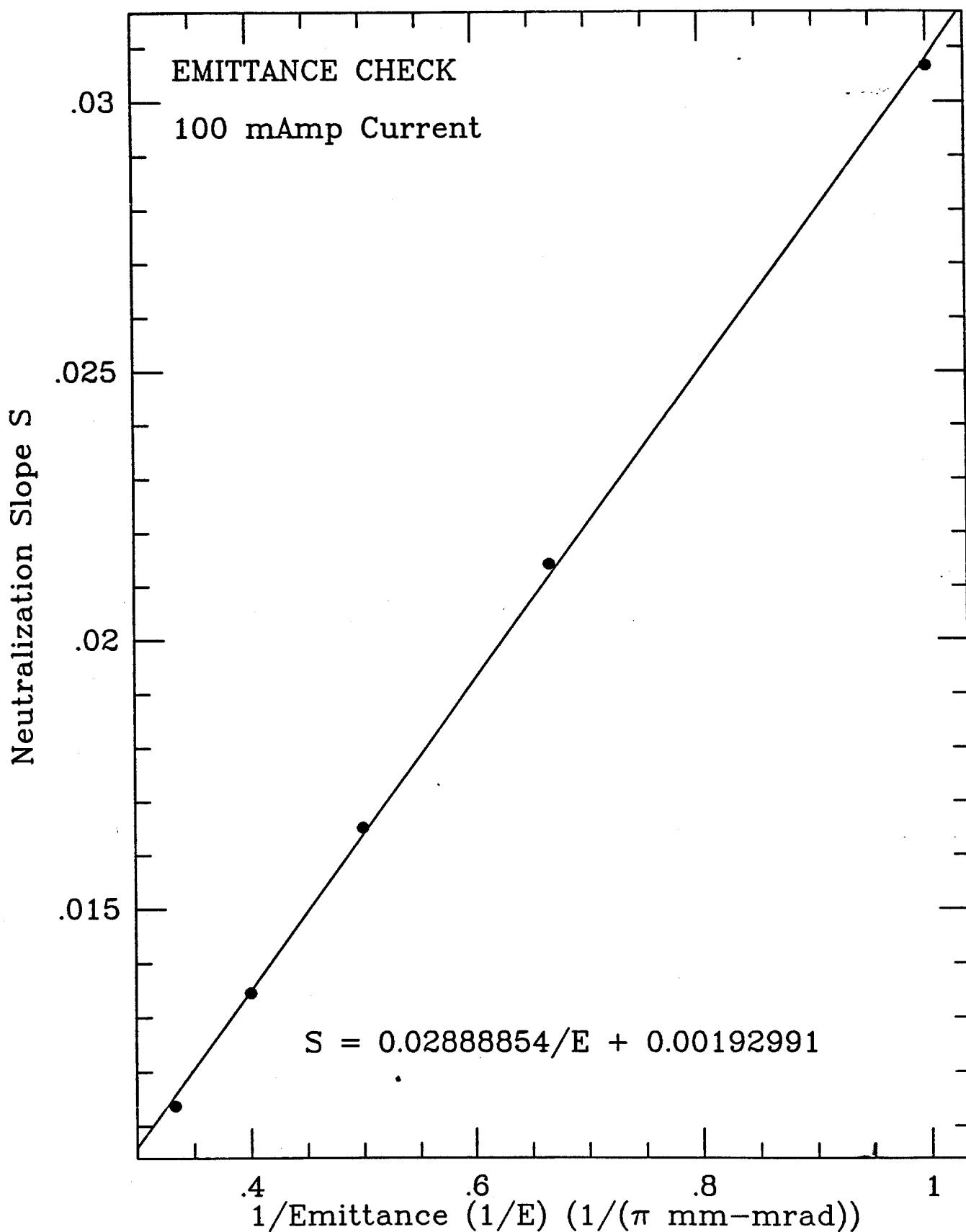


Figure 36